

INDIAN STATISTICAL INSTITUTE

Analysis I : B. Stat 1st year
Mid Semester Examination: 2025-26
10.09.2025

Maximum Marks: 30

Maximum Time: 2 hrs.

Answer all questions but maximum you can score is 30.

(1) Does the series $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ converge? Justify your answer. [3]

(2) Let $x_1 = 1$, $x_2 = 2$, and

$$x_n = \frac{1}{2}(x_{n-1} + x_{n-2}), \quad n \geq 3.$$

Prove that $\{x_n\}$ converges and then find $\lim_{n \rightarrow \infty} x_n$. [5]

(3) Let $1 \leq x_1 \leq x_2 \leq 2$, and

$$x_{n+2} = \sqrt{x_n x_{n+1}}, \quad n \geq 3.$$

(a) Prove that $\frac{x_{n+1}}{x_n} \geq \frac{1}{2}$, for all $n \in \mathbb{N}$. [2]

(b) Prove that $\{x_n\}$ converges. [3]

(4) Find all possible values of $p \in \mathbb{R}$, such that the series $\sum_{n=1}^{\infty} \frac{\cos n}{n^p}$ is:

(a) Absolutely convergent. [2]

(b) Conditionally convergent. [4]

(c) Divergent. [2]

(Hint: You can assume the fact that $\lim_{n \rightarrow \infty} \cos n$, does not exist.)

[P.T.O]