INDIAN STATISTICAL INSTITUTE

BACK PAPER EXAMINATION
M.TECH(CS/CrS-I) & M. MATH II YEAR

AUTOMATA THEORY, LANGUAGES, COMPUTATION AND COMPLEXITY

Date: 02.01.2026 Maximum marks: 100 Duration: 3.0 hours.

The paper contains 105 marks. Answer as much as you can, the maximum you can score is 100.

1. (a) Give example of an infinite language $A \subset \{0,1\}^*$ which is regular.
   (b) Give description of a finite automata M_A, such that $L(M_A) = A$.
   (c) Formally prove that $L(M_A) = A$.
   (d) Give a regular expression α_A, such that $L(\alpha_A) = A$.

[3+7+8+2=20]

2. (a) Give example of a language $B \subseteq \{0,1\}^n$ which is not regular.
   (b) Formally argue why there can be no DFA which accepts B. [3+7=10]

3. For any $x \in \Sigma^*$, define

$$\mathsf{SUF}_x = \{zx : z \in \Sigma^*\}.$$

   (a) Consider $\Sigma = \{0,1\}$, construct a NFA with 5 states which accepts SUF_{0011}.
   (b) For $y, z \in \Sigma^*$, let $\mathsf{overlap}(y,z)$ be the longest string which is a suffix of y and a prefix of z. For $x \in \Sigma^*$, we define a relation $\approx_x$ where for $y, z \in \Sigma^*$

$$y \approx_x z \overset{\text{def}}{\iff} \mathsf{overlap}(y,x) = \mathsf{overlap}(z,x).$$

   Show that for any $x \in \Sigma^*$, $\approx_x$ is a Myhill Nerode relation on Σ^* for SUF_x.
   (c) Prove that the minimal DFA accepting SUF_x has exactly $|x|+1$ states.

[3+7+5=15]

4. (a) Give a context free grammar G_C which generates the language $C = \{a^n b^m : m \neq n\}$.
   (b) Represent the grammar G_C in Chomsky's Normal Form.
   (c) Formally describe a push down automata which accepts the language C by empty stack.

[5+5+5=15]

5. (a) A Turing Machine M has only two states q and f, where q is the start state and f the final state. Its input symbols are from $\Sigma = \{0, 1\}$ and the blank symbol B is the only other tape symbol. The transition functions are $\delta(q, 0) = (q, 0, R)$, $\delta(q, B) = (f, 1, R)$. What is the language accepted by M? Formally prove your answer.

   (b) Consider the language

$$D = \{(M, k) : M \text{ is a TM which accepts at least one string of length } k\}.$$

   Show that D is not recursive. You may assume that $\mathsf{MP} = \{(M, x) : x \in L(M)\}$ is not recursive.

[(3+5)+7=15]

6. (a) Define the classes P, NP and coNP.

   (b) Prove that P $\subseteq$ NP.

   (c) Prove that NP is closed under intersection.

   (d) Prove that P $\subseteq$ NP $\cap$ coNP

[6+3+3+3=15]

7. (a) Let L_1 be NP complete and $\overline{L_1} \in$ NP. Prove that for all $L \in$ NP, $\overline{L} \in$ NP.

   (b) Consider the languages SAT and $\mathsf{7SAT}$ described below:

$$\begin{aligned} \mathsf{SAT} &= \{\phi : \phi \text{ is a satisfiable Boolean formula in CNF}\}. \\ \mathsf{7SAT} &= \{\phi : \phi \text{ is Boolean formula in CNF with at least 7 satisfying assignments}\}. \end{aligned}$$

   Show that $\mathsf{SAT} \leq_p \mathsf{7SAT}$.

[5+10=15]