INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2025-2026

Computer Vision

M.Tech.(Computer Science)

Date: 12.09.2025 **Full Marks: 40** **Time: 2 Hours**

Answer any **four** questions. All questions carry equal marks.

1. (a) Show that the camera calibration problem for transforming 3D world points to 2D image points can be formulated as a least squares problem.

   (b) Derive the intrinsic matrix of a camera by solving the least squares problem.

   (c) Considering homogeneous coordinates, show that inferring 3D world points from 2D image points can be solved in a least squares sense.

   [4+3+3=10]

2. (a) Prove that when a camera views a fronto-parallel plane from an unknown distance, the relation between the normalized camera coordinates and the 2D points on the plane is a similarity transformation.

   (b) Consider a set of 2D positions $\{w_i\}$ on the surface of the plane and the corresponding 2D image positions $\{x_i\}$ in the image. What is the minimum number of pairs of points required to learn the rotation matrix Ω, translation vector τ, and scaling factor ρ of the similarity transformation?

   (c) Prove that the translation vector τ of the similarity transformation between points $\{x_i\}$ and $\{w_i\}$ is given by

   $$\tau = \mu_x - \rho\Omega\mu_w$$

   where μ_x and μ_w are the means of the points $\{x_i\}$ and $\{w_i\}$, respectively.

   (d) Prove that the problem of estimating the rotation matrix Ω of the similarity transformation can be formulated as Orthogonal Procrustes problem.

   [3+1+3+3=10]

3. (a) A camera with intrinsic matrix Λ views a plane from an arbitrary viewpoint. It takes a 2D image considering extrinsic parameters $\Omega = \text{I}$, $\tau = 0$, and then moves to a new position with $\Omega = \hat{\Omega}$, $\tau = 0$ and takes a second image. Show that the homography relating these two images is given by

   $$\Phi = \Lambda\hat{\Omega}\Lambda^{-1}.$$

   (b) Suppose two cameras view the same planar scene. The one-to-one mapping from the positions on the plane to the positions in the first and the second cameras can be described by two homographies T_1 and T_2, respectively. Show that the mapping T_3 from the first image to the second image is given by

   $$T_3 = T_2T_1^{-1}.$$

(c) The 2D point x_2 is created by a rotating point x_1 using the rotation matrix Ω_1 and then translating it by the translation vector τ_1 so that

$$x_2 = \Omega_1 x_1 + \tau_1.$$

Find the parameters Ω_2 and τ_2 of the inverse transformation

$$x_1 = \Omega_2 x_2 + \tau_2$$

in terms of the original parameters Ω_1 and τ_1.

(d) Prove that the differentiation of the output of a convolution, of a signal with a filter, can be achieved by convolving the signal with the derivative of that filter.

[3+2+3+2=10]

4. (a) Define epipole and epipolar line.

(b) Show that the essential matrix can capture the geometric relationship between two normalized cameras.

(c) Derive the rotation matrix Ω and translation vector τ between two normalized cameras, when the essential matrix between them is known.

[2+4+(2+2)=10]

5. Prove that the diffusion of grey value from one pixel to other pixels can be modelled by the following heat diffusion equation:

$$\frac{\partial I(x, y; \sigma)}{\partial \sigma} = \sigma \Delta I(x, y; \sigma),$$

where Δ represents the Laplacian, $I(x, y; \sigma)$ is the image defined in the 3D scale space, as follows:

$$I(x, y; \sigma) = I(x, y) * G(x, y; \sigma),$$

$I(x, y)$ is the input image, $*$ denotes the convolution operation, and

$$G(x, y; \sigma) = \frac{1}{2\pi\sigma^2} \exp\left\{\frac{-(x^2 + y^2)}{2\sigma^2}\right\}.$$

[10]

6. (a) Define central moment and principal axis of an image.

(b) What is bi-linear interpolation?

(c) Compute the mutual information between two co-registered images A and B, where

$$A = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & 1 \end{pmatrix}.$$

[(2+2)+2+4=10]