INDIAN STATISTICAL INSTITUTE

END SEMESTER EXAMINATION 2025-26

MSQE I

STATISTICS

Answer as many questions as you can. The maximum mark is **60** (including assignments).

Time: 3 HRS.

DATE: 24.11.2025

1. Determine the maximum likelihood estimator for the range $\theta_2 - \theta_1$ from a random sample drawn from a population that is Uniform on (θ_1, θ_2). Find the bias of the estimator.

[5+3]

2. (a) Let X_1 and X_2 be independent and identically distributed Poisson random variables with parameter λ. Show that $X_1 - X_2$ is not a sufficient statistic for λ.

   (b) Let $X_1, X_2, \ldots, X_n$ be *i.i.d.* normal random variables with $X_i \sim N(\theta, \theta^2)$, $i = 1, 2,..n$. Find a minimal sufficient statistic for θ and determine whether it is complete or not. [4+4]

3. Suppose that we have two independent random samples: $X_1, \ldots, X_n$ are exponential (θ), and $Y_1, \ldots, Y_m$ are exponential (μ).

   (a) Find the LRT statistic of $H_0: \theta = \mu$ versus $H_1: \theta \neq \mu$

   (b) Show that the test statistic in part (a) can be based on $T = \frac{\sum X_i}{\sum X_i + \sum Y_j}$

   (c) Find the distribution of T when H_0 is true.

[4+2+4]

4. A pharmaceutical company is testing a drug for a specific side effect. They test n = 50 groups of patients, with k = 10 patients in each group. The probability that any single patient experiences the side effect is p. The company's report only states the number of groups, Y , in which at least one patient experienced the side effect.

   (a) Let p^* be the probability that a group has at least one side effect. Express p^* in terms of p and k. [3]

   (b) The report states that Y = 19 groups had the side effect. Find the Maximum Likelihood Estimate of p. [4]

5. Let $X \sim U(0, \theta)$. It is decided to test $H_0 : \theta = 2$ vs $H_1 : \theta \neq 2$ on the basis of a random sample of size 1 using the following rule: Reject H_0 if either $x_1 \leq 0.1$ or $x_1 \geq 1.9$. Find the probability of type I error and the power function.

[3+4]

P.T.O.

6. Let $X_1, X_2, \ldots, X_n$ be i.i.d. with density

$$f(x;\theta) = \theta x^{\theta-1}, \; 0 < x < 1, \theta > 0$$

Define the statistic $T_n = -\frac{1}{n}\sum_{i=1}^{n} ln\, X_i$

(a) Show that $E(T_n) = \frac{1}{\theta}$ and $Var(T_n) = \frac{1}{n\theta^2}$

(b) Find the UMVUE of $\frac{1}{\theta}$

[2+2+6]

7. Suppose that $X_1, \ldots, X_n$ form a random sample from the normal distribution with unknown mean μ and known variance σ^2. How large a random sample must be taken in order that there will be a confidence interval for μ with confidence coefficient 0.95 and length less than 0.01σ?

[4]

8. Asit and Tista have opened a coffee shop and are interested in understanding the true average customer satisfaction rating μ. They assume individual ratings follow a Normal distribution with unknown mean μ and known variance σ^2. From their past market research, they believe μ itself has a Normal distribution with mean μ_0 and variance τ^2. Since they expect to receive variable numbers of ratings at different times, they want to derive a general formula for the posterior distribution of μ based on n observed ratings, $x_1, \ldots, x_n$.

(a) Derive the posterior distribution of μ, and find the posterior mean and variance.

(b) After several weeks, Asit collects ratings from 5 customers: 7.2, 8.5, 7.8, 8.0, and 7.4. Assuming their prior parameters remains $\mu_0 = 7.0$, $\tau^2 = 1.5$, and variance $\sigma^2 = 1$ still holds, calculate the posterior mean and variance.

(c) Find the 95% Highest Posterior Density credible interval for μ, and interpret what this interval means about the average customer satisfaction in their coffee shop.

[4+2+3]

9. Assignments. [7]