INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination:(2025-2026)

M.TECH (CS) I YEAR

Subject Name: Quantum Computation

Maximum Marks: 30 Duration: 2 hours Date: 11.09.2025

Answer any three of the following four questions

1. a) Consider the following linear operator A acting on a two dimensional Hilbert space.;

$$A = a_o I + a_x \sigma_x + a_y \sigma_y + a_z \sigma_z$$

where σ's are usual Pauli matrices and a_0, a_x, a_y, a_z are real numbers. Under what condition A is a density matrix.
b) Show that for every the density operator D with $D^2 = D$, there exists a unit vector $|\psi\rangle$. such that the following relation holds.

$$D = |\psi\rangle\langle\psi|$$

c) Let the initial density matrix of a qubit is $\frac{1}{2}(I + \frac{1}{3}\sigma_x + \frac{1}{2}\sigma_y)$. If spin measurement is performed along z-axis, what is the probability for the spin up result?
4 + 3 + 3

2. a) Let there is a cloning machine that can clone the following orthogonal states $|0\rangle$ and $|1\rangle$. Show that the machine will not be able to clone the following state $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$.

b) Consider a Swap operator U_s which acts in the following way;

$$U_s|\psi\rangle \otimes |\phi\rangle = |\phi\rangle \otimes |\psi\rangle$$

for all possible states $|\psi\rangle$, $|\phi\rangle$. Then show that U_s can not be expressed as

$$U_s = U_1 \otimes U_2$$

where U_1 and U_2 are acting on particle 1 and particle 2 respectively.

c) Describe the realization of the swap operation (gate) by using C-Not gates.

4 + 3 + 3

3. a) Let Alice and Bob share the following state:

$$|\psi>_{AB}= \frac{1}{\sqrt{5}}(|0\rangle_A \otimes |0\rangle_B + \frac{2}{\sqrt{5}}|1\rangle_A \otimes |1\rangle_B)$$

where $|0\rangle$ and $|1\rangle$ are eigen states of σ_z.
(i) Show that the state can not be written in product form.
ii) Find the density matrix of the subsystem on Bob's side.
(ii) Derive the probability of conclusive teleportation of an unknown state by using this state. How many bits will be required for conclusive teleportation?

2 + 2 + 6

3. i) Consider a two qubits pure state $|\psi\rangle_{12}$ and four unitary operators $\{U_i, i = 1,2,3,4\}$ acting on the first particle. What is the necessary condition so that the four states $\{U_i \otimes I|\psi\rangle_{12}, i = 1,2,3,4\}$ form an orthogonal set?
ii) Find a two qubits state $|\psi\rangle_{12}$ for which $\{U_i \otimes I|\psi\rangle_{12}, i = 1,2,3,4\}$ form an orthogonal set.
iii) Discuss how quantum super dense coding can be realized using the $|\psi\rangle_{12}$ in (ii) .

3 + 3 + 4