Indian Statistical Institute
First Semester Examination: 2025-2026 (Regular)
Course Name: B.Stat II, Subject Name: Physics I

Date: 27.11.2025 Duration: 3 hours

Answer as many questions as you can. Use of Calculator is allowed. Maximum Marks: 50

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1. (a) Let, a particle of mass m moves inside a bowl, surface of which is given by the equation: $z = \frac{1}{2}\alpha(x^2 + y^2)$. where α is a constant.

Determine the Lagrangian of the particle in simplest form.

b) A double pendulum consists of two point masses, both having mass m and attached by massless springs of length, l, as shown in the Figure 1.

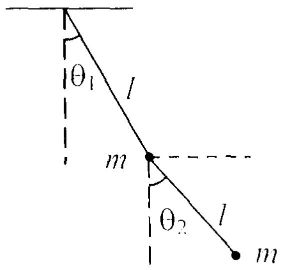

Figure 1

Establish the expression of the kinetic energy of the double pendulum. [5 + 5]

2. (a) Use the Hamiltonian equations to find the differential equations for planetary motion and prove that aerial velocity is constant.

(b) A charged particle (let, electron) is moving under the influence of point nucleus. Find the orbit of the particle and the time period in the case of an elliptical orbit. Consider that a is the semi-major axis of the ellipse, m is the mass of the electron, k is the Coulomb's constant, z be the atomic number of the nucleus, e is the charge of the electron, and T is the time period. [5 + 5]

3. (a) Water is heated at a constant pressure of 0.7 MPa. The boiling point is 164.97°C. The initial temperature of water is 0°C. The latent heat of evaporation is 2066.3 kJ/kg. Find the increase of entropy of water, if the final state is steam.

(b) Calculate the entropy change of the universe as a result of the following processes:

(i) A copper block of 600 g mass and with c_p of 150 J/K at 100°C is placed in a lake at 8°C.

(ii) The same block, at 8°C, is dropped from a height of 100 m into the lake.

(iii) Two such blocks, at 100 and 0°C, are joined together. [5 + 5]

4. (a) Write down the basic differences between Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics.

(b) Prove that Maxwell's law of distribution of x-component of velocity v_x among the molecules of an ideal gas is represented by

$$n(v_x)dv_x = N\left(\frac{m}{2\pi kT}\right)^{1/2} e^{-mv_x^2/2kT} dv_x$$

where N is the total number of molecules, m is the mass of a single molecule, k is the Boltzmann constant and T is the absolute temperature. [5 + 5]

5. (a) In the temperature range between 0°C and 100°C a particular system, maintained at constant volume, has a heat capacity, $C_v = A + 2BT$

where $A = 0.014$ J/K and $B = 4.2 \times 10^{-4}$ J/K^2

A heat reservoir at 0°C and a reversible work source are available. What is the maximum amount of work that can be transferred to the reversible work source as the system is cooled from 100°C to the temperature of the reservoir?

(b) Prove that according to Maxwell's law of distribution, the number of molecules of an ideal gas in equilibrium at temperature T, whose magnitude of momenta lie in the range between p and $p + dp$ is given by

$$n(p)dp = \frac{4\pi N}{(2\pi mkT)^{3/2}} p^2 e^{-p^2/2mkT} dp$$

where N is the total number of molecules, m is the mass of a single molecule, k is the Boltzmann constant and T is the absolute temperature. [5 + 5]

6. (a) Using the Maxwell-Boltzmann distribution law, prove that:

$$C_v = \frac{3}{2}R$$

where C_v is the molar specific heat of the gas at constant volume and R is the gas constant for 1 mole.

(b) Consider that a gas has only two particles, A and B. Each one of them can be in one of the three quantum states, $g_r = 1, 2, 3$. Find in a tabular form the possible number of microstates of the gas according to the Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics. [5 + 5]