INDIAN STATISTICAL INSTITUTE

M.Stat. (2^{nd} Year)

Subject: Time Series Analysis

Semestral Examination (2025-26)

Date: 18/11/2025 | Full Marks: 50 | Duration: 2 hours 30 minutes

Attempt All Questions

1. Consider the stationary time series given by the equation

$$X_t + 0.5X_{t-1} = Z_t - 2Z_{t-1},$$

where $\{Z_t\} \sim \text{WN}(0, 2)$.

(a) Write X_t as a linear process in $\{Z_t\}$.

(b) Comment about causality and invertability of the series.

(c) Derive ACVF $\gamma(h)$ $(h \geq 0)$ of the series.

[4+2+4]

2. Define PACF. Explain how it is used to test if a time series is AR(p) or, not. **[3+5]**

3. (a) Define Periodogram $I_n(\omega_k)$. Show that it equals $\sum_{|h|<n} e^{-ih\omega_k}\ \hat{\gamma}(h)$, where n is data size, ω_k is multiple of $\frac{2\pi}{n}$ and $\hat{\gamma}$ is sample ACVF.

(b) If f is spectral density of some absolutely summable ACVF γ, show $f(\lambda) \geq 0$, f is even and $\gamma(h) = \int_{-\pi}^{\pi} e^{ih\lambda} f(\lambda)\ d\lambda$.

[(2+6)+8]

4. (a) Show that linear predictor $P_n(X_n|X_{n-1}, \ldots, X_1, X_0)$ is unique for a weakly stationary time series $\{X_t\}$.

(b) State Wold Decomposition Theorem. Using this show that if a stationary time series is q-correlated, then $\exists$ a WN series $\{Z_t\}$ with which it is MA(q).

[7+(3+6)]

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