Indian Statistical Institute

Semestral Examination 2025

Programme - MSQE-II
Course - Econometric Methods II

Time - 3 hours | Date - Nov 19, 2025 | Total Marks: 50

Attempt all the questions. Notations are as per class notes. No marks will be awarded for calculation mistakes.

1. Identify whether the following processes are trend stationary or unit root or none of the two. Answer with proper justification.

   (a) $y_t = \alpha + \beta t + \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2}$

   (b) $y_t = y_{t-1} + \varepsilon_t - 2\varepsilon_{t-1} + \varepsilon_{t-2}$

   (c) $y_t = \alpha + \beta t + u_t$, where $u_t = u_{t-1} + \varepsilon_t + 2\varepsilon_{t-1}$

   (d) $y_t = 0.4y_{t-1} + 0.6y_{t-2} + \varepsilon_t + 2\varepsilon_{t-1} - 3\varepsilon_{t-2}$

   (e) $(1 - L)y_t = u_t$, where $u_t = u_{t-1} + \varepsilon_t$

   For all the above cases $\varepsilon_t \sim WN(0, \sigma^2)$.

   [$1 \times 5 = 5$]

2. Let $y_t = \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \varepsilon_t$, where $\varepsilon_t \sim WN(0, \sigma^2)$. Show that the above process can be presented as

$$\Delta y_t = (\rho - 1)y_{t-1} + \sum_{j=1}^{p-1} \delta_j \Delta y_{t-j} + \varepsilon_t$$

   where ρ and δ_j's are functions of $\phi_1, \cdots, \phi_p$.

   [5]

3. (a) Define a trend-stationary process and a difference-stationary process with example.

   (b) Show that for a difference stationary process, removing trend is not enough to make the process stationary.

   (c) Show that a trend stationary process and a difference-stationary process are some special cases of the following process:

$$y_t = \alpha + \beta t + u_t$$

   where $u_t = \phi_1 u_{t-1} + \cdots + \phi_p u_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$ and is invertible and $\varepsilon_t \sim N(0, \sigma^2)$.

   [2+2+6=10]

4. Let $\mathbf{y}_t = (y_{1t}, y_{2t})'$ be a bivariate time series such that

$$y_{1t} = \alpha + \beta y_{2t} + u_{1t}, \text{ and } y_{2t} = y_{2(t-1)} + u_{2t}$$

where u_{1t} and u_{2t} are $WN(0, \sigma^2)$ and are uncorrelated with each other. Show that $\mathbf{y}_t$ is cointegrated.

[10]

5. Let $\mathbf{y}_t$ be a stationary bivariate AR(1) process given by

$$\mathbf{y}_t = \alpha + \Phi \mathbf{y}_{t-1} = \varepsilon_t,$$

where $\Phi = \begin{pmatrix} 0.5 & 0.3 \\ 0.3 & 0.5 \end{pmatrix}$, $\alpha = (1.5, 2.5)^T$, and $\varepsilon_t \overset{iid}{\sim} N(0, \Omega)$, where $\Omega = \begin{pmatrix} 4 & 1 \\ 1 & 9 \end{pmatrix}$

(a) Find the impulse response function (IRF) for 1st row and 2nd column at h=5, i.e., find $\dfrac{\partial y_{1,t+5}}{\partial \varepsilon_{2,t}}$.

(b) Find the autocorrelation between Y_{1t} and $Y_{2(t-1)}$.

[6+4=10]

6. You have panel data for two districts (A and B) observed for three years. The outcome variable is *rice production* (Y_{it}) and the explanatory variable is *rainfall* (X_{it}).

District	Year	X_{it} (Rainfall)	Y_{it} (Rice production)
A	1	10	48
A	2	12	52
A	3	11	50
B	1	8	38
B	2	9	40
B	3	10	41

Consider the following model:

$$Y_{it} = \alpha_i + \beta X_{it} + u_{it},$$

where α_i represents the district-specific fixed effect.

(a) Using the mean subtraction approach, compute the estimate $\hat{\beta}_{FE}$ of the slope in the model above.

(b) Compute the district fixed effects $\hat{\alpha}_A$ and $\hat{\alpha}_B$.

(c) For District A in Year 2, compute the fitted value $\hat{Y}_{A2}$ and the residual.

[6+2+2=10]