End-Semester Examination, M. Math (Academic Year: 2025-2026)
Topology-I
November 17, 2025

Maximum Marks: 60 Time: 3 hours

Provide answers with complete justifications. It is not an open-book exam.

1. (a) Is the product of two Lindelof spaces Lindelof? If not give an example. (2)
   (b) Is every well-ordered set normal in order topology? If not then give an example. (2)
   (c) Is $\mathbb{R}^{\omega}$ normal in the product topology? In the uniform topology? (2)
   (d) State the **Urysohn metrization theorem**. (2)
   (e) Let $p \in \mathbb{S}^1 \times \mathbb{S}^1$, then what is $\pi_1((\mathbb{S}^1 \times \mathbb{S}^1, p)) \cong$? (2)
   (f) Is $\mathbb{D}^2$ and $\mathbb{S}^1$ are of the same homotopy type or not? (2)
2. Show that a connected normal space having more than one point is uncountable. (8)
3. Give a direct proof of the Urysohn lemma for a metric space (X, d).(*Hint:* consider $f(x) = \frac{d(x,A)}{d(x,A)+d(x,B)}$.) (5)
4. State the Baire Category Theorem. Use it to show that the set of rational numbers is not a Baire space. (6)
5. Show that if X has a countable basis $\{B_n\}$, then every basis C for X contains a countable basis for X. (10)
6. Let $X = \mathbb{R}^3 - \{n \text{ straight lines passing through origin}\}$, where $n > 0$ is a positive integer. Show that X deformation retracts onto $\mathbb{S}^2 - \{2n \text{ points}\}$. (6)
7. Let $h : \mathbb{S}^1 \to \mathbb{S}^1$ be a nullhomotopic map. Show that there exist $x, y \in \mathbb{S}^1$ such that $h(x) = x, h(y) = -y$. (8)
8. Recall that $\mathbb{RP}^n = \mathbb{S}^n/(x \sim -x)$. Prove that the quotient map $\mathbb{S}^n \to \mathbb{RP}^n$ is a covering map. (10)
9. Consider the covering map $p : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}^2 - \{(0,0)\}$ defined as $p(x,t) := te^{2\pi i x}$. Let $f : [0,1] \to \mathbb{R}^2 - \{(0,0)\}$ be a path defined as $f(s) = (2 - s, 0)$. Find a lifting of the path f. (5)