Indian Statistical Institute
M Tech (QROR) I Year: 2025-26
Semester I
End-Semester Examination
Operations Research I

Date: 19/11/2025 Time: 3 hours Full Marks: 100

Note: (i) Question no. 1 is compulsory. Answer only five questions out of the rest.

(ii) Calculators are allowed.

1. Explain the following terms:
   (a) Degeneracy
   (b) Balking
   (c) VED analysis
   (d) Discrete Event System Simulation
   (e) Safety stock

[3+3+3+3+3]=15

2. (a) Prove that if the feasible region $K \subseteq R^n$ of a linear programming problem is a convex polyhedron and an optimal solution exists, then there is at least one extreme point of K that is optimal.

   (b) Convert the following problem into a standard LP problem.

   $Min\ |x| + |y| + |z|\ ,\ subject\ to\ x + y \leq 1, 2x + z = 3.$

   (c) Let $u, v\ and\ w$ be distinct vectors of a vector space V. Show that if $\{u, v, w\}$ is a basis for V, then $\{u + v + w, v + w, w\}$ is also a basis for V.

   (d) Determine which of the following are bases for $P_2(R)$.
      a. $\{-1 - x + 2x^2, 2 + x - 2x^2, 1 - 2x + 4x^2\}$
      b. $\{1 - 2x - 2x^2, -2 + 3x - x^2, 1 - x + 6x^2\}$
      c. $\{-1 + 2x + 4x^2, 3 - 4x - 10x^2, -2 - 5x - 6x^2\}$
      d. $\{1 + 2x - x^2, 4 - 2x + x^2, -1 + 18x - 9x^2\}$

[5+5+3+4]=17

3. (a) Let $V = M_{2x2}(R), W_1 = \{\begin{pmatrix} a & b \\ c & a \end{pmatrix} \in V: a, b, c \in R$, and $W_2 = \begin{pmatrix} 0 & a \\ -a & b \end{pmatrix} \in R: a, b \in R$. Prove that W_1 and W_2 are subspaces of V, and find the dimensions of W_1, W_2, $W_1 \cap W_2$ and $W_1 + W_2$. R is a set of real numbers.

(b) Determine which of the following sets are bases for R^3.

i. $\{(1,0,-1),(2,5,1),(0,-4,3)\}$
ii. $\{(2,-4,1),(0,3,-1),(6,0,-1)\}$
iii. $\{(1,2,-1),(1,0,2),(2,1,1)\}$
iv. $\{(-1,3,1),(2,-4,-3),(-3,8,2)\}$
v. $\{(1,-3,-2),(-3,1,3),(-2,-10,-2)\}$

(c) The set of solutions to the system of linear equations

$$x_1 - 2x_2 + x_3 = 0$$
$$2x_1 - 3x_2 + x_3 = 0$$

is a subspace of R^3. Find the basis for this subspace. **[8+5+4]=17**

4. The manager of a company is planning to undertake a new project from the client. He is ready now to start the project, and wishes to finish in 60 days. He identifies the 8 separate activities, their precedence relationship and their respective duration in days, which are shown as follows:

Activity	Precedence Relationship	Normal Time (days)	Crash Time (days)	Normal Cost (Rs.)	Crash Cost (Rs.)
A	-	6	5	14,500	18,000
B	A	11	8	15,000	21,000
C	B	18	16	28,000	30,400
D	A	16	15	22,000	25,000
E	B	9	8	15,000	16,000
F	C, D	12	9	19,500	24,000
G	E, F	15	13	30,000	42,000
H	G	5	4	9,000	9,500

(a) Construct a project network for the above data.

(b) Find all the paths and path lengths through this project network. Which of these paths form a critical path?

(c) Find the latest start time and earliest finish time for each activity.

(d) Find the minimum possible cost for the project to finish in 60 days. (Assume that for each activity there is a single linear, continuous function between the crash duration and normal duration points). **[2+3+2+10]=17**

(b) A famous director is working on four upcoming Hindi films, each requiring one leading actor. The estimated "casting cost" (lower = better fit) for each actor–film combination is given below:

Actor / Film	A	B	C	D
X1	9	2	7	8
X2	6	4	3	7
X3	5	8	1	8
X4	7	6	9	4

(i) Formulate this as an assignment problem.

(ii) Find the optimal assignment of actors to films using the Hungarian Method.

(iii) Determine the minimum total casting cost. **[10+(2+3+2)]=17**

5. A logistics firm must deliver goods from 3 warehouses (W_1, W_2, W_3) to 4 retail outlets (R_1–R_4). Each shipment can go by one of transport from Truck or Rail, and the cost per ton (₹) for each route is shown below. Choose the cheaper mode for each route and find the minimum total transportation cost.

From\To	R_1	R_2	R_3	R_4	Supply
W_1	8 / 6	5/9	10/7	9/8	30
W_2	7/5	6/6	9/5	8/7	40
W_3	4/4	7/6	6/7	5/5	50
Demand	20	35	25	40	

(a) Pick the cheaper mode (Truck or Rail) for each cell.

(b) Find the optimal shipment plan and minimum total cost using the NWCR method and MODI method and compare the solution.

(c) State whether the solution is unique or has multiple optima. **[6+6+5=17]**

6. (a) Stating the assumptions derive the Production order quantity (POQ) model.

(b) Explain Poisson process. Derive the expression of the waiting time distribution and the performance measures in the steady state for a single server queue under the assumption of Poisson arrival and exponential distribution of service time with infinite queue length, infinite population and FCFS queue discipline.

(c) Patients arrive at a clinic every 12 minutes (Poisson arrival) and the nurse can serve at the rate of 8 per hour (exponential). What is the expected number of patients in the queue? What is the expected waiting time in the clinic? What is the probability that the clinic is not having any patient? **[4+10+3]=17**

7. (a) A retailer wants to reduce its ordering costs by placing larger quantity orders using the price-break order quantity schedule below. Considering the cost of fuel, it is estimated that the cost will be about Rs. 800 to pick up an order. The carrying cost is 12% of the price of the item, and there is an annual demand of 1,750 units. Determine the optimal order quantity according to the EOQ model with quantity discounts. What is the resulting total cost per year? With this order quantity, how many orders need to be placed per year? What is the time interval between orders?

Order Quantity (Units)	Price/unit (Rs)
1 to 399	60
400 to 799	42
800 or more	30