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EXPLORING CONFLICTS AND PROTESTS THROUGH THE LENS OF GAME THEORY

by

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Declaration of Authorship

I, Puja Mukherjee, declare that this thesis titled, 'Exploring Conflicts and Protests Through The Lens of Game Theory' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at the Indian Statistical Institute.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
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Signed: Puja Mukherjee

Date: 16. 01. 2026

“Knowledge will make us free.”

Swami Vivekananda

ABSTRACT

This dissertation aims to study conflicts and protests using a game-theoretic setup. It contains six chapters. Chapter 1 is the introduction. Chapter 2 gives a brief review of the related literature. Chapter 3 explores conflicts in the presence of revenge and third party intervention using a game theoretic setup. In chapter 4, I propose a revenge-capability function to study the strategic behaviour of the conflicting parties. In chapter 5, I develop a signaling game between the government and protesters to study the phenomenon of protests in the presence of an external shock like the pandemic. Chapter 6 concludes the dissertation.

Chapter 3 aims to study conflicts in the presence of revenge and third-party interventions using a game theoretic setup where the intervention decision of the third party is endogeneous. This model explores parametric restrictions under which a third party decides to intervene (either as an ally of one of the conflicting parties or as an 'idealist' aiming to reduce overall conflict levels) and its repercussions on associated conflict levels. This chapter also presents narrative evidences of some real-life conflicts that amply exhibit the two forces of third-party intervention and revenge.

In chapter 4, I propose a revenge-capability function that endogenously incorporates the incapacitation effect and study the strategic behaviour of the conflicting parties. Using a two-period game of conflict this chapter tries to show how desire and capabilities of the combatants to exact revenge can influence the intensity of the conflict. This chapter shows the following: how the strategies of the conflicting parties are influenced by the different effects of revenge; how the stronger combatant is in a favourable position in the conflict and can prevent its opponent from going into second period conflict out of revenge; when the combatants are equally strong the intensity of the conflict starts falling with time. It also lays out some real-life conflicts and existing empirical work to support the results.

In chapter 5, I develop a signaling game where the protesters' type is imperfectly observed by the government to study when it will be optimal for the protesters to protest in response to a government action and for the government to use a repression strategy when there is an external shock like the pandemic. It shows the following; how the virus spread influences the strategies of the players; how the intensity of protests changes with the level of the virus spread; compares the no-pandemic equilibria with the pandemic equilibria and lastly analyses the parametric conditions under which different separating and pooling equilibria holds.

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Chapter 1

Introduction

"It is a behavior situation in which each player's best choice depends on the action he expects the other to take, which he knows depends, in turn, on the other's expectation of his own. This interdependence of expectations is what distinguishes a game of strategy from a game of chance or a game of skill."

Thomas Schelling in *The Strategy of Conflict*

Conflicts can be defined in many ways. The simplest definition of conflict is a situation where inadmissible differences in opinions, interests and expectations arise between individuals or groups or more specifically between agents. A more technical definition of conflict is; a situation involving competition such that the competitive influence is evident and the conflicting parties or agents know and take into account the fact that they have a noticeable influence on each others' payoffs. The analysis of conflict requires numerous numbers of interdependent behaviours and game theory is considered as a natural inheritor of marginalist method. The spirit of game theoretic reasoning is that agents before taking an action, rationally evaluate the effects of their choices on the behaviour of their opponents (Osborne, 2004 ; Gibbons, 1992). This dissertation aims to explore conflicts and protests through the lens of game theory.

A fundamental inquiry in international relations and political economy revolves around identifying the factors that influence the beginning and progression of conflict among various entities such as countries, political factions and ethnic or religious communities.

Thucydides[1] in his examination of the Peloponnesian War, attributed the root causes of conflict to factors like greed, honor and fear. He argued that the Peloponnesian War became unavoidable because both sides perceived it as inevitable and were unwilling to give up the advantage of striking first to the other (Kagan 2004). This perspective on conflict reminds us of the 'Hobbesian' view or the spiral model[2]. The dynamic implication of this view of conflict is quite transparent: when Group A acts aggressively, Group B interprets this as a sign of aggression from Group A and responds aggressively in turn (Jervis 1976 ; Kydd 1997). Additionally, unless Group A comprehensively recognizes that Group B's aggression stems solely from Group A's actions, it will perceive Group B's response as further evidence of Group B's aggression. Consequently, this cycle leads to an escalation of the conflict.

Following the philosophy of Hobbes, Baliga and Sjöström (2012) have shown that when actions are strategic complements, uncertainty can generate fear which can trigger conflicts. Acemoglu and Wolitzky's (2014) model of cycles of conflict is based on classical idea that the main cause of a conflict is often distrust and misperceived aggression. The cycle of conflict can be explained using different perspectives. However, the root cause of violence escalation is attributed to honor which leads to conflicts out of revenge. Gershenson and Grossman's (2000) two-party model emphasizes the significance of intrinsic and economic values that rival parties place on political dominance, in explaining the initiation and continuation of intrastate conflicts. Esteban and Ray (2006) in their ethnic conflict model showed that an increase in wealth disparity among members of the same ethnic group could intensify inter-ethnic conflicts since it is less expensive to mobilize the impoverished, while the affluent possess greater resources to support such mobilization. Dasgupta and Kanbur (2005) showed how separatism lays the groundwork for communal conflict by creating motivations for all the individuals to support the expropriation of the other communities. Dasgupta and Kanbur (2007) showed that an increase in economic mobility of certain groups could potentially worsen class tensions rather than reduce them.

Extensive research seeks to explain conflict as stemming from incomplete information, specifically the uncertainty that parties have about their rivals' preferences, capabilities,

[1]Thucydides was an ancient Greek historian, considered as the greatest of all. He was an Athenian general who commanded a fleet in the Peloponnesian war. He wrote the *History of the Peloponnesian War* which is the first documented political and moral assessment of a nation's wartime strategies. It provides a detailed chronological narrative that covers the reason behind the conflict, the nature of the warring factions and the technical details of the warfare, incorporating events where he was directly involved. The reference here is Thomas Hobbes' complete translation of the book *The Peloponnesian War* by Thucydides, published in 1989.

[2]The Hobbesian Trap (named after Thomas Hobbes (1588-1679), an English philosopher renowned for his work in political science) is also referred to as Schelling's Dilemma (following Schelling, 1960). This theory elucidates that preemptive strikes can occur between two groups out of mutual fear of an impending attack and a drive for self-preservation. The intensification of this fear has the potential to escalate, triggering an arms race.

as well as their previous behaviors and future plans. For example, in the arms race model, each government may assign a low yet significant probability that the adversary is genuinely aggressive, making acquiring arms the best strategy. This uncertainty increases the perceived advantage of defensive actions that is acquiring new weaponry. However, because each player is aware that their opponent is reasoning in a similar manner, a multiplier effect occurs, leading to a cycle of increasingly negative expectations that tend towards an inferior Pareto equilibrium (Schelling (1960)). Baliga and Sjöström (2004) demonstrate the existence of 'cheap-talk' equilibria where the discourse provides meaningful information that can effectively diminish the likelihood of an arms race. They show that an effective communication can significantly reduce the probability of an arms race, practically nullifying it.'

Several empirical studies show that the root source of civil conflict is the abundance of natural resources[3], calling it the 'resource curse'. The 'resource curse' literature posits that the abundance of natural resources increases the likelihood of civil conflict (Soysa, 2000; Wick and Bulte, 2006). Munshi (2017) used a game theoretic setup to examine the potential consequences of alleged nexus between government and industry when allocating public resources especially when the local community, the resource's original users, might resist with armed force.

Ever since the end of Cold War, there has been a growing necessity not just to comprehend the roots and triggers of conflicts within states, but also to grasp methods of controlling or even stopping them (Regan, 1996). While understanding the underlying causes is crucial for dealing with and resolving these conflicts, it is equally essential to have a grasp of the prevailing dynamics between the warring factions and any intervening third parties. This understanding is pivotal for assessing the prospects of achieving successful outcomes, especially in conflicts where more direct intervention is necessary and where there is no room for a negotiated settlement or where there is no existing "peace to maintain". Depending upon the degrees of aggression and hostility there are various kinds of intervention and peace-keeping missions[4]. Morgenthau (1967, p. 430) states, "All nations will continue to be guided in their decisions to intervene... by what they regard as their respective national interests." The third party could potentially

[3]Using the data on wars from 1960-99, Collier and Hoeffler (2004) investigated the root causes of civil war. They tested the 'greed' theory against the 'grievance' theory. In short, the 'greed' theory centers on the potential to finance a rebellion whereas the 'grievance' theory focuses on inequality, ethnic and religious divisions and political repression. They found minimal evidence for grievance as a cause of conflict, while the greed model provided a better explanation of conflict occurrence. Their findings confirmed that resource-rich nations are more prone to conflict.

[4]Doyle and Sambanis (2000), classifies international peace-building operations into four categories: monitoring and observer missions, traditional peacekeeping, multidimensional peacekeeping, and peace enforcement. The first three typically involve supporting negotiated agreements and require the consent of the host government, while the fourth type may or may not require such consent.

benefit from greater access to resources and trade, improved national security, ethical achievements, and geopolitical advantage[5].

1.1 Motivation:

In real life, winning a conflict sometimes does not end the conflict. Revenge motivations can stay and provide momentum to the conflict, thus leading to further escalation of the conflict. This is known as the value effect or vengeance effect of revenge. However, the presence of revenge can lead to de-escalation of the conflict out of self-deterrence and sometimes retaliation out of revenge is not possible if the combatant is incapacitated. For example; the Empire of Japan's capitulation followed the first use of atomic bombs by the US, marking the end of the war.

Alongside revenge, another factor that can lead to escalation or de-escalation of conflict is third party intervention in a conflict. Third-party intervention can lead to escalation or de-escalation of conflict levels. As found in the conflict literature, third-party interventions can mainly be of two broad kinds – either a third party intervenes as an ally of one of the conflicting parties, or it intervenes as an 'idealist' with an intention to reduce overall level of conflict.

Real-world conflicts rarely remain isolated; they are typically marked by third-party intervention and violent revengeful interactions between the conflicting parties. This statement can be explained from a few real-life examples of conflicts[6].

The conflict between Catholics and Protestants in North Ireland began in late 1960s. The period after 1968 has been called the 'Troubles'. British soldiers came in 1969 to bring order to society, but unfortunately took sides and the discrimination against Catholics went on. Terrorism and murder were carried out both by extreme Catholics and extreme Protestants.

The 2019-2020 Persian Gulf crisis started in May 2018 when United States withdrew from the Joint Comprehensive Plan of Action (JCOPA) nuclear deal, which reinstated the sanctions against Iran. Military tensions between Iran and the United States escalated in 2019 amid a series of confrontations involving the US, Iran, and Saudi Arabia. In this conflict Saudi Arabia supported US.

Yemen has been embroiled in a complex and multi-sided civil war since the end of 2014. The conflict in Yemen is between the UN-recognised government of Mansour Hadi,

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[5] https://iep.utm.edu/polreal/

[6] The following conflicts are discussed in details in the 'Narrative Evidence' section of each chapters.

supported by Saudi Arabia and the United Arab Emirates (UAE), and the Iranian-backed Houthis.

The Israeli-Palestinian conflict dates back to the end of the nineteenth century. The State of Israel was created on May 14th 1948, leading to the first Arab-Israeli War. The war ended in 1949 with Israel's victory, and the territory was divided into 3 parts: the State of Israel, the West Bank (of the Jordan River), and the Gaza Strip. Over the years, tensions rose in the region, particularly between Israel and Egypt, Jordan, and Syria. The Israel-Palestine conflict can be described as a complicated vicious cycle of violence that does not seem to end.

This dissertation aims to explore the strategic behaviour of the conflicting parties and its impact on the intensity of conflict in the presence of factors like revenge and third-party intervention. It discusses about it in the following two chapters;

The third chapter explores conflicts in the presence of revenge and third-party intervention using a game theoretic setup. This chapter shows that the conflicting players are better off exacting revenge on each other, irrespective of whether the third party has intervened or not (either as an ally of one of them or as an 'idealist'). Given this, the third party intervenes as an ally only when the (strategic) value of the resource, if its ally wins, is sufficiently high. While the third party intervenes as an idealist only if its intervention can significantly reduce resultant conflict levels.

In the fourth chapter I introduce a revenge-capability function that endogenously incorporates the incapacitation effect and then using game theory I study the strategic behaviour of the conflicting parties. This chapter models a revengeful conflict in a two period game-theoretic setup where a combatant's desire to go into a second period conflict out of revenge depends on its capability to do so.

The fifth chapter is an attempt to game-theoretically model protests that the world witnessed during the pandemic.

A protest is a public demonstration of objection and dissent aimed at an action or idea. Protesters gather with the intention of voicing their viewpoints to sway public opinion or government policy. Throughout history, protests have posed a challenge to governments, whether in democratic, non-democratic, or semi-democratic systems. When confronted with a protest, governments face a decision: they can either accommodate the protesters or deploy security forces (such as protest police or riot police) to suppress the demonstration. History shows that suppressing protests can escalate violence and radicalize the public, potentially leading to civil disobedience and triggering a chain of violence that may ultimately undermine the ruling government.

Now, consider if an external shock such as a pandemic occurs, how will the actions of protesters and the government be affected? During a pandemic, organizing a protest and suppressing demonstrators both become expensive for the respective parties involved. Therefore, the key question becomes: as the severity of the virus fluctuates, when is the optimal time for protesters to rally against government actions, and for the government to implement repression strategies? This chapter seeks to address this inquiry especially considering the instances from India. During the COVID-19 pandemic, India witnessed two noteworthy protests, the anti-CAA (Citizenship Amendment Act, 2019) protest and the Farmers' protest (2020).

On 11th December 2019, the Parliament of India passed the Citizenship (Amendment) Act, 2019 (CAA). The CAA, 2019 amended the Citizenship Act of 1955 by allowing accelerated Indian citizenship to persecuted religious minorities from Afghanistan, Bangladesh and Pakistan who are Hindus, Sikhs, Buddhists, Jains, Parsis or Christians and they arrived in India before December 2014. This act excludes Muslims. Under the Indian Law, this was the first time that religion was overtly used as a criterion for citizenship. With the announcement of this act India witnessed a massive number of protests throughout the country criticizing the amendment.

The 2020–2021 Indian farmers' protest was a protest against the three farm acts that were passed by the Parliament of India on September 2020. The Indian Central Government introduced three agricultural reform bills on June 2020. The three laws were: Farmers' Produce Trade and Commerce (Promotion and Facilitation) Act, 2020; Farmers (Empowerment and Protection) Agreement on Price Assurance and Farm Services Act, 2020; and Essential Commodities (Amendment) Act, 2020.

The outcome of these two protests were different. The anti-CAA protest was taken-off by the protesters whereas the farmers' protest continued until the government acquiesced to the farmers' demands. In the fifth chapter I develop a signaling game between government and protesters where the protesters type is imperfectly observed by the government to study the phenomenon of protests in the presence of an external shock like the pandemic.

The chapter-wise summary is given below;

1.2 Chapter 3

Most real-world conflicts are characterised by the presence of third-party interventions as well as prolonged and revengeful violent interactions between the conflicting parties.

However, most of the conflict literature studies the impact of each of these forces – third-party intervention and revenge motivations – on the conflict, in isolation from each other. This chapter attempts to fill in this gap and aims to explore the impact of these two forces acting in conjunction with each other on a conflict situation.

This chapter augments Amegashie and Runkel (2012) setup by introducing an endogenous third-party intervention that can either be as an ally of one of the combatants (Chang, Potter and Sanders, 2007) or as an 'idealist'. It explores the conditions and the parametric restrictions under which Amegashie and Runkel's (2012) findings (of the paradox holding or not) may or may not still hold good. It analyses the subgame perfect Nash equilibrium (SPNE) of the game and finds out the parametric conditions under which a third-party decides to intervene or not.

1.3 Chapter 4

Given the existing literature, revenge has been modelled (in a game-theoretic setup) as an increasing function of the opponent's conflict investment of the previous period (Amegashie and Runkel 2012; Liang, Chen, and Siquiera 2020). But in reality, the motive of a combatant to go into a second period conflict out of revenge depends on its military capability to do so. This model fills in this gap and analyses the strategic behaviour of the combatants and its impact on the intensity of the conflict.

In Amegashie and Runkel's (2012) formulation, the revenge function in the current period is an increasing function of the opponent's conflict investment in the previous period. In this chapter I modify this and introduce what we call a 'revenge-capability' function where the amount of revenge this period not only depends on the opponent's conflict investments of the previous period, but also on its own resource capability that is available with it, given destruction suffered in the previous period. The results in this two-period game resonates with Dixit's (1987) results. It shows that the combatant who can partially incapacitate its opponent in the first period conflict is in a favourable position in the conflict and thus will increase its first period effort and prevent its opponent from going into second period conflict out of revenge. This model shows that when both the combatants are strong then the conflict can gain momentum in the beginning but as a the conflict continues and the destruction increases it reduces the conflicting parties' incentive to further retaliate and thus the intensity of the conflict starts falling with time. It also shows how self-deterrence effect influences the first period conflict and incapacitation effect influences the second period conflict.

1.4 Chapter 5

In this chapter, I develop a signaling game where the protesters' type is imperfectly observed by the government to study how the players (government and protesters) will interact in a protest during a pandemic. First, the benchmark case of no-pandemic has been discussed and then it has been compared with the pandemic case. The pandemic is incorporated by using a random variable which captures the virus spread and is common knowledge to both the players. This model shows how the virus spread influences the strategies of the players and the intensity of the protests. It shows that at the initial stages of the pandemic the government might prefer to repress the protesters and since the pandemic also increases the cost to protest for the protesters, the protesters might prefer to accept government's proposal and not protest. However, as the pandemic also imposes an additional cost on the government, repression of protests becomes costly for the government. Thus, as virus spread increases the fighting efforts of both the government and the protesters starts falling and when the virus spread increases beyond a certain threshold the government is better-off accommodating the protesters.

Chapter 2

Literature Review

This dissertation employs game theory to analyse the strategic underpinnings of conflicts in the presence of revenge and third-party intervention. This chapter provides a comprehensive review of existing literature that explores the same.

Revenge is believed to be the major cause of continuing conflict other than major cause of original conflict. Chagnon (1988) from his studies of the Yanomamö people in the Amazon Rainforest, empirically showed that the major cause of conflict escalation was retaliatory killings.

However, contrary to expectations, the fear of retaliation or revenge can also serve as a reason for the weakening of conflict. Amegashie and Runkel (2012) calls it the 'paradox of revenge'. They have explained the paradox of revenge by characterising two effects - the 'value of revenge' effect which is the benefit of exacting revenge (which puts an upward pressure on overall conflict level) and the 'self-deterrence' effect which is fear of the opponent's desire to exact revenge or retaliate (which puts a downward pressure on overall conflict level). They have constructed examples where the equilibrium is such that the self-deterrence effect paradoxically exceeds the value effect and thereby decreases the aggregate conflict investments below that exerted when there is no revenge.

Liang et al. (2020) re-examined the vengeance effect, self-deterrence effect and the paradox of revenge in a defender-attacker scenario. As per their framework, the defender (and only the defender) who was attacked in the the first period takes revenge on the attacker in the ensuing period.

Chen and Siqueira (2022) in their defender-attacker model showed that the presence of revenge can change the nature of conflict in a number of ways. They showed that when the value of revenge is significantly high, then the presence of revenge can limit the level of conflict or it can deter it altogether.

Jaegar and Paserman (2008) studied the dynamics of violence in the Israel-Palestine conflict during the four years of the Second Intifada using daily frequency data. They empirically examined whether violence against each of the conflicting parties, that is Israel and Palestine, can affect the intensity and extent of each side's reaction. They found that Israel significantly reacts to Palestinian violence against them whereas Palestine's actions are not significantly related to Israeli violence against them.

Rehman et al. (2017) empirically examined the efficacy of various counterinsurgency policies employed in Pakistan. They examined the vengeance effect, deterrence effect and incapacitation effect under various counterinsurgency policies. As per their results, peace accords have no momentous effect on violence, military operations exacerbate violence indicating a dominant vengeance effect. Whereas, the National Action Plan, operation Zarb-e-Azb [1], generated a significant incapacitation effect thus leading to a significant reduction in violence.

Regan (1998) conducted a study to evaluate the third-party intervention strategies and relative success rate, using data on all intra-state conflicts since 1944. The results showed that under certain conditions, third-party interventions in intra-state conflict can expedite the end of the violent facet of the conflict.

Regan (2002) assumed that the third parties plays the role of an mediator or 'conflict manager', and their main attempt is to limit hostilities. Similarly, Siqueira (2003) also assumed that the short run objective of an intervener is to reduce or weaken the existing level of conflict. Chang, Potter and Sanders (2007) called these approaches as 'liberal or idealist'.

Chang, Potter and Sanders (2007) developed a sequential game in which they endogenized the third-party intervention by considering a scenario in which the third party's welfare depended on the outcome of a territorial dispute between two factions. In their setting, the third-party acted as a 'selfish' agent, who is interested in maximising its own payoff.

Amegashie and Kutsoati (2007), similarly examined the endogeneity of third-party intervention in an intra-state conflict, by portraying the third party as a 'benevolent social planner', who maximises the weighted sum of the welfare of the combatants and non-combatant when deciding the optimal level of intervention.

[1] Pakistan's military and political leadership learnt that taking control of an area from terrorists does not deter or reduce attacks. Rational and well-connected terrorists can just move from one region to another in response to counterinsurgency attacks. Thus, the government of Pakistan implemented the Nation Action Plan, a comprehensive counterinsurgency strategy across the country, established by the government in January 2015 as a counterterrorism strategy across the whole country to complement the ongoing military offensive (Operation Zarb-e-Azb) in North Western Pakistan. It was considered as a major coordinated state retaliation following the Peshawar school attack in December 2014.

Beviá and Corchón (2010) analyzed a model of war where the players are rational and there is complete information. As per their model a war can be avoided in many cases in the absence of binding agreements, if one player transfers money to the other. Their paper explores the possibility of such transfer agreements in a four stage game with finitely lived and fully-informed players. However, Garfinkel and Skaperdas (2000) showed in their model that war can emerge as an equilibrium outcome when the future holds long-term compounding rewards. In their model each player chose between guns and butter. First, they considered a static one-period game where peaceful settlement is the preferred end-result, and then they considered a two-period model to show their primary result of the analysis.

Thyne (2006) examined the impact of interstate signals on the probability of civil war onset. The author utilizes a bargaining model to argue that costly signals do not make civil war more likely. This is because they enable the government and opposition to modify their bargaining positions without resorting to violence, thus avoiding the costs of fighting. Whereas cheap signals can create distortions in internal negotiations, making conflict a more probable outcome.

Chapter 3

Conflict with Third-party Intervention and Revenge: A Game-Theoretic Exploration

An extended version of this chapter has been published as:
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3.1 Introduction

"Peace is the only battle worth waging." - Albert Camus

'Should a third party's intervention decision be different when warring factions have a desire to seek revenge relative to when they do not have such a desire?', question Amegashie and Runkel in the concluding lines of their paper, 'The Paradox of Revenge in Conflicts' (2012). This chapter is a direct attempt to answer this. It tries to explore two potent forces - third party intervention and revenge - acting in conjunction with each other, on a conflict. Not only do we analyse what happens to conflict levels, but also shed light on the third party's decision, as to whether or not it intervenes in the first place. Hence we endogenize the intervention decision of the third party and thereafter explore its repercussions on ongoing, potentially revengeful, conflict situations.

Real world conflicts are often characterised by the presence of both third party interventions as well as revenge motivations of the conflicting parties. Third parties are often assumed to mediate and reduce conflict (see Regan (1996) and Siqueira (2003), for example). However, third party intervention may also be in the form of acting as an ally of one of the parties and hence exacerbating conflict (see Chang, Potter and Sanders (2007), for example). Similarly, revenge motivations are also believed to potentially increase and prolong conflicts. Though, as Amegashie and Runkel (2012), show, it is possible that conflicts actually fall when players are revenge-motivated. In short, therefore, when both the forces of third party interventions and revenge motivations, act on conflictual players, it is a priori, quite hard to predict how conflict levels and overall welfare of the players are affected.

For example, consider the territorial dispute over Jammu and Kashmir in the Indian subcontinent between India and Pakistan for over seventy years[1]. Over the years both India and Pakistan have chosen to make Jammu and Kashmir the cornerstone of their respective identities. In their maximalist versions, Kashmir is claimed to be India's *atoot ang* (integral part) and Pakistan's *shah rag* (jugular vein). And it has witnessed some of the most unending bloodshed of modern times. But not only is it a tale of revengeful neighbours perpetuating conflict, but also third party interventions. China has been an ally of Pakistan in most of the diplomatic wars and in its conflict against India[2]. We discuss several other real life conflicts in the Narrative Evidence (Section 3.4) that drag on for years being subject to revenge motivations of the opponents as well as interventions by third parties (like the Yemeni conflict, Persian Gulf crisis, etc.). In short, any real life conflict situation is likely to be under both forces acting simultaneously - intervention by third parties (pushing down or fanning conflict) and revenge (again ameliorating or exacerbating conflict). This chapter intends to bring these very potent influences acting on a conflict into closer conversation with each other.

Let us briefly look at the concepts of revenge and third party intervention before formally turning to our model.

3.1.1 Revenge

Revenge is a strong emotional trigger that mobilises people into action. As Michael McCollough, evolutionary psychologist of University of Miami, states it as 'it's this very

[1]Very briefly, the origin of the problem dates back to the independence of the Indian subcontinent from Britain and its partition into India and Pakistan in 1947. Till then Jammu and Kashmir was one of the largest princely states of the subcontinent under the 'indirect rule' of the British. With the lapse of the British 'paramountcy', the princely states were asked to accede to one of the two 'Dominions', India or Pakistan. The first Indo-Pak war over Jammu and Kashmir took place in January 1949, thereafter in 1965 and 1972, and numerous infiltrations and skirmishes in between and till recently. Thus was the beginning of an incessant story of reprisals and violence. See Munshi (2013) for details.

[2]https://en.wikipedia.org/wiki/Indo-Pakistani_wars_and_conflicts#Background

pervasive experience in human lives, people from every society understand the idea of getting angry and wanting to hurt someone who has harmed him/her'[3] *BBC*, 2017. Revenge not only drives up crime rates but it also plays a pivotal role in politics. According to an article in the Washington Post, as of October 15, 2015, Donald Trump's victory came has a result of 'revenge of working-class whites...who felt abandoned by a rapidly globalising economy'[4]. Many studies have illustrated that exacting revenge brings satisfaction to the person who has suffered previous harm. De-Quervain et al. (2004) conducted a study which supported the hypothesis that people derived satisfaction from punishing norm violations, and the anticipated satisfaction from punishing defectors was reflected by the activation in the dorsal striatum.

Revenge is also believed to be the major cause of continuing conflict other than major cause of original conflict. Paradoxically, the prospect of retaliation may act as a deterrent, ultimately facilitating the de-escalation of conflict. Amegashie and Runkel (2012) calls it the 'paradox of revenge'.

In short, revenge considerations can both drive up and down, overall conflict levels, depending on the strengths of the value that the players place on revenge and the fear of retaliation from the opponent to their own conflict investments.

3.1.2 Third party intervention

Like revenge, another factor that can lead to escalation or de-escalation of conflict is third party intervention in a conflict. To begin with, third party intervention is an endogenous decision on the part of the third party, and hence the question arises as to what makes a country intervene in some other dispute that does not directly concern it. According to Morgenthau (1967), countries choose to intervene when national interests are at stake. Regan (1998) calls it the 'paradigm of realism'. Blechman (1995) and Carment and James (1995) posit that ethical issues and domestic policies play a crucial role in the decision to intervene. The general type of third party intervention is military subsidy, as considered by Siqueira (2003). Increase in subsidy increases the likelihood that the ally gains or maintains possession of the territory.

In short, third party intervention again can lead to an increase or a decrease in conflict, depending on nature of intervention and a host of other circumstantial factors.

[3] https://www.bbc.com/future/article/20170403-the-hidden-upsides-of-revenge.(accessed on 12.12.2020)

[4] https://www.washingtonpost.com/news/wonk/wp/2015/10/15/i-asked-psychologists-to-analyze-trump-supporters-this-is-what-i-learned. (accessed on 12.12.2020)

3.1.3 The Model

This chapter attempts to put these two strands of literature into closer conversation with each other. We augment the Amegashie and Runkel (2012) set-up by introducing an endogenous third party intervention that can either be as an ally of one of the combatants or as an 'idealist' (drawing upon Chang, Potter and Sanders, 2007). Hence, we explore conditions and parametric restrictions under which Amegashie and Runkel's (2012) findings may or may not still hold good. There are, in fact, cases where any form of third party intervention can alter the paradox of revenge that may have been in effect without any intervention.

Amegashie and Runkel (2012) considered a two-period game of conflict between two players competing over a given resource. They also incorporated the idea of revenge, where each player wants to exact revenge for the previous destruction suffered. Hence, say in time t, players makes conflict investments. Then in time $t+1$, they exact revenge on each other. We augment this game, by introducing another period before the two conflicting players make their conflict investments - when a third player decides whether or not to intervene in the subsequent conflict-revenge game of the existing conflicting parties. Hence in time $t-1$, a third party decides whether or not to intervene. Thereafter, the Amegashie and Runkel's (2012) two-period conflict game is played out.

As found in the conflict literature, third party interventions can mainly be of two broad kinds - either a third party intervenes as an ally of one of the conflicting parties, or it intervenes as an 'idealist' intending to reduce overall conflict levels thereby reducing loss of human lives and damage that conflict causes. We assume that whether a third party acts as an ally or an idealist is dictated by history, norms etc., and is exogenous for purposes of the present analysis[5]. Hence in Section 3.2 we assume that the third party decides to intervene or not as an ally of one of the conflictual players, and thereafter, the players engage in conflict investments and revenge. We analyse the SPNE of the game and find out parametric restrictions under which the third party decides to intervene as an ally or it decides to not intervene at all.

In Section 3.3 we assume that the third party decides to intervene or not as an 'idealist', and thereafter, the players engage in conflict investments and revenge. Once again, we analyse the parametric restrictions under which the third party decides to intervene as an idealist or it decides to not intervene at all, in any SPNE. We closely look at and

[5]For example, to take the case of Saudi intervention in the Yemeni conflict - given Sunni-dominated Hadi government and given Saudi Arabia being a Sunni power of the area, it is possibly beyond doubt that, if at all, Saudi Arabia would definitely intervene as an ally of the Hadi-government rather than as an 'idealist' trying to reduce conflict between Shia-majority Houthis and Sunni-majority Hadi government. See Section (3.4.1) for details.

compare the associated equilibrium conflict levels and welfare of the players in all the resulting games.

We find that the conflicting players are better off exacting revenge on each other, irrespective of whether the third party has intervened or not (either as an ally of one of them or as an 'idealist'). Given this, the third party intervenes as an ally only when the (strategic) value of the resource, if its ally wins, is sufficiently high. While the third party intervenes as an idealist only if its intervention can significantly reduce resultant conflict levels.

The rest of the chapter is organised as follows: Section 3.2 lays out the model and the results when the third party intervenes as an ally of one of the sides. Section 3.3 lays down the model and results when the third party is an idealist. Section 3.4 discusses some real world conflicts where both third party intervention and revenge motivations of the players seem to play important roles. Section 3.5 concludes with a discussion on future research questions.

3.2 Third party Intervening as an Ally

Two players, $i = A, B$, are engaged in combat against each other over two periods of time, $t = 2, 3$. There is a third party, denoted by T. As explored in Chang, Potter and Sanders (2007), a third party can intervene either as a ally of one of the combatants or as an idealist who wants to reduce overall conflict levels. In this section, let us assume that it is acting as an ally and supports A[6]. In the next section, we explore the possibility of the third party being an idealist and wanting to reduce overall conflict levels. After the third party decides whether or not he wants to intervene as an ally of A, the players compete for a resource in period 2, and whoever wins it, gets to keep it. However in period 3, the players may decide to exact revenge for the atrocities suffered at the hands of its opponent in period 2. Thereafter, the game ends.

Figure 3.1 depicts the game. The sequence of moves is as follows: First, the third party decides whether it will intervene as an ally of A or not. In the next period, A, B simultaneously decide on conflict investments. In the third period, both players simultaneously decide on whether they will exact revenge on each other or not[7]. Hence,

[6]For example, think of China supporting Pakistan in its conflict against India or the Saudi Arabia supporting Yemen in its conflict against the Houthis (who is supported by Iran again), and so on.

[7]Revenge is played out by both the players even if only one of them chooses to take revenge - its opponent is automatically drawn into taking revenge.

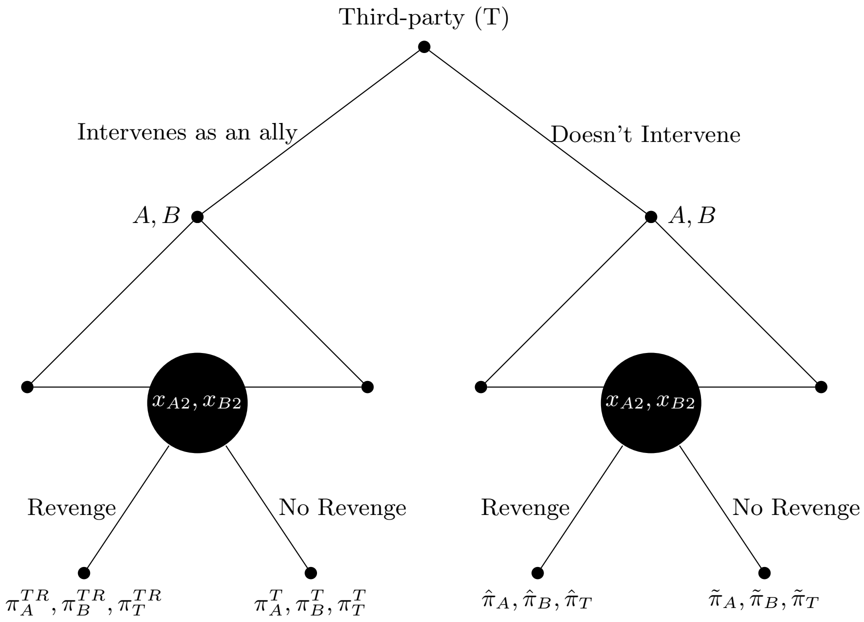

FIGURE 3.1: Third-party intervening (or not) as an ally in the presence (or absence) of revenge motivations

the different possibilities in this case and the corresponding payoffs (listed as payoff of A, payoff of B, payoff of T) are as follows[8]:

(i) The third party doesn't intervene and there is no revenge $(\tilde{\pi}_A, \tilde{\pi}_B, \tilde{\pi}_T)$; (See section 3.2.1 below)

(ii) The third party doesn't intervene and A, B exact revenge $(\hat{\pi}_A, \hat{\pi}_B, \hat{\pi}_T)$; (See section 3.2.2 below)

(iii) The third party intervenes but there is no revenge $(\pi_A^T, \pi_B^T, \pi_T^T)$; (See section 3.2.3 below)

(iv) The third party intervenes and there is revenge $(\pi_A^{TR}, \pi_B^{TR}, \pi_T^{TR})$; (See section 3.2.4 below)

Since it is a game of perfect information, we use backward induction to find the Subgame Perfect Nash Equilibrium (SPNE).

[8]Notice the slight digression from convention where the usual order of payoffs follows the order in which players move. Here however, though the third party moves first, its payoff is listed last.

Subgame Equilibrium - Conflict and Revenge

In this section, we analyse the subgames where players A, B are engaged in conflict and revenge (or not). In the next section we look at the SPNE of the entire game.

Let player i's valuation of the resource that they compete over, be $V_i, i = A, B$. Let x_{it} be the 'conflict investment' of player $i, i = A, B$ in period $t, t = 2, 3$. Let the probability with which player i wins the resource in period t be given by the standard ratio-form Tullock (1980) contest success function as follows:

$$P_{it} = \frac{x_{it}}{x_{it} + x_{jt}}, i, j = A, B, i \neq j.$$

Also for simplicity, assume that the cost of effort is linear and the marginal cost is normalised to 1. Hence the utility functions of the players in period 2 are given as follows:

$$\pi_{i2} = P_{i2}V_i - x_{i2}, \quad \text{where}, i = A, B$$

Now let us turn to revenge. Laying down the setting of revenge of Amegashie and Runkel (2012), we have revenge functions, R's in period 3, which depend on conflict investments of one's opponent in period 2. For example, R_A would be as follows:

$$\begin{aligned} (i) \quad & R_A(x_{B2}) > 0, \text{if } \ x_{B2} > 0, \\ (ii) \quad & R_A(x_{B2}) = 0, \text{if } \ x_{B2} = 0, \\ (iii) \quad & R'_A(x_{B2}) > 0. \end{aligned}$$

Hence 'revenge' investment of player A in period 3 is positive whenever conflict investment of player B in period 2 is positive. Moreover, it is increasing in the opponent's conflict investment.

3.2.1 No third party intervention and no revenge

As a benchmark case, we first lay out the basic model without third party intervention and without the possibility of revenge[9]. In the absence of revenge, there is no third period revenge investments. Letting all variables in the benchmark case be denoted by tilde (without digressing from the Amegashie and Runkel (2012) paper), we have $\tilde{x}_{A3} = \tilde{x}_{B3} = 0$. Let $\tilde{X}_3$ be the aggregate revenge investments in period 3. Hence

[9] This is essentially from the section 'Equilibrium without Revenge' in the Amegashie and Runkel (2012) paper (pg. 317) with $\eta = 1; j = A, k = B$, with the difference that the time subscript 1 has become 2 here and 2 has become 3 here, given we have added an additional stage to the game where the third party, player T, moves.

$\tilde{X}_3 = \tilde{x}_{A3} + \tilde{x}_{B3} = 0$. In period 2, the utility function of player $i, j = A, B, i \neq j$, is given by the following:

$$\pi_{i2} = P_{i2}V_i - x_{i2}. \tag{3.1}$$

Maximising w.r.t. x_{i2}, and assuming an interior optimum, we arrive at the equilibrium levels of conflict investments, $\tilde{x}_{A2}, \tilde{x}_{B2}$. Hence we can find the aggregate conflict investment in period 2, $\tilde{X}_2 = \tilde{x}_{A2} + \tilde{x}_{B2}$. Summing over two periods (assuming away discounting), we can arrive at the aggregate level of investments in violence (conflict and revenge), $\tilde{X} = \tilde{X}_2 + \tilde{X}_3 = \tilde{X}_2 + 0 = \tilde{X}_2$. We arrive at the following values:

$$\tilde{x}_{i2} = \frac{V_i^2 V_j}{(V_A + V_B)^2}, \quad i, j = A, B, \quad i \neq j;$$
$$\tilde{X}_2 = \tilde{X} = \frac{V_A V_B}{(V_A + V_B)}.$$

3.2.2 No third party intervention but revenge

With minor alterations, this part is essentially adopted from Amegashie and Runkel (2012), pg. 318 - 319, and hence is not discussed in details here[10]. A very brief outline is as follows: there is conflict in period 3 owing to revenge of the players (in addition to that in period 2 over the resource). It can be calculated that the equilibrium levels of period 3 conflict investments, $\hat{x}_{A3}$ and $\hat{x}_{B3}$ and the aggregate level of third period conflict $\hat{X}_3 = \hat{x}_{A3} + \hat{x}_{B3}$ as follows:

$$\hat{x}_{i3} = \frac{R_i^2 R_j}{(R_A + R_B)^2}, \quad i, j = A, B, \quad i \neq j; \tag{3.2}$$
$$\hat{X}_3 = \frac{R_A R_B}{(R_A + R_B)}. \tag{3.3}$$

Conflict efforts in the second period fall in this case relative to the first benchmark case of no-revenge, no-third-party as laid down in 3.2.1. Amegashie and Runkel (2012) call this the 'self-deterrence effect' of revenge. Self-deterrence effect is the effect where each combatant anticipates its effort in period 2 will cause its opponent to exact revenge in period 3 as a result each player reduces its effort in the conflict over the resource in the second period.

We now turn to the case where a third party acts as an ally of player A but the combatants are not revengeful.

[10] Please refer to Appendix A.0.1.

3.2.3 Third party intervention as an ally of A but no revenge

The third party intervenes as an ally of A only in the second period conflict against B for the resource. Since there is no revenge by assumption, there is no third period conflict. The third party, T, provides military subsidies, M, to party A. Using Chang, Potter and Sanders' (2007) cost reduction function, $s = \frac{1}{(1+M)^{\theta}}$, where θ measures the degree of effectiveness with which a dollar of subsidy reduces player A's unit cost of arming and $0 < \theta < 1$. The utility functions of A and B in this case are as follows:

$$\pi_{A2} = P_{A2}V_A - \frac{1}{(1+M)^{\theta}}x_{A2}; \tag{3.4}$$

$$\pi_{B2} = P_{B2}V_B - x_{B2}. \tag{3.5}$$

Maximising the utility functions w.r.t x_{i2} in the simultaneous game and assuming an interior solution we get the equilibrium levels of conflict investments x^T_{A2}, x^T_{B2} and the aggregate level of conflict in period 2, $X_T = x^T_{A2} + x^T_{B2}$, as follows:

$$x^T_{A2} = \frac{V_A^2 V_B (1+M)^{2\theta}}{(V_A(1+M)^{\theta} + V_B)^2}; \tag{3.6}$$

$$x^T_{B2} = \frac{V_B^2 V_A (1+M)^{\theta}}{(V_A(1+M)^{\theta} + V_B)^2}; \tag{3.7}$$

$$X_T = X_{T2} = \frac{V_A V_B (1+M)^{\theta}}{(V_A(1+M)^{\theta} + V_B)}. \tag{3.8}$$

Observation 3.1. As expected, $\tilde{X} < X_T$, i.e., an intervention by a third party, acting as an ally, increases the overall conflict investments (by making conflict less costly for one of the players).

Let us now turn to the contribution of this chapter - to analyse the model with both a third party intervening (as an ally in this case) and revenge motivations of the players. The third party, can, for example, provide military subsidies (M) like arms and ammunition and thus reduce the cost of conflict of its ally by enhancing its military efficiency and potentially increasing its conflict efforts. Chang, Potter and Sanders (2007) used the example of Soviet Union's intervention with military assistance during the Cold War on behalf of Afghanistan's ruling Marxist government to explain the third-party intervention as an ally of one of the combatants.

3.2.4 Third party intervention as an ally of A and revenge

For this subsection, the third party intervenes in the conflict as an ally of player A in the second period. In the third period, when players exact revenge on each other, the third party does not intervene (by assumption). Proceeding like before, the second period payoff functions of players A and B are as follows:

$$\pi_{A2} = \frac{x_{A2}}{x_{A2}+x_{B2}}V_A - \frac{1}{(1+M)^\theta}x_{A2} + \hat{\pi}_{A3}; \tag{3.9}$$

$$\pi_{B2} = \frac{x_{B2}}{x_{A2}+x_{B2}}V_B - x_{B2} + \hat{\pi}_{B3}. \tag{3.10}$$

Maximising the payoff functions w.r.t x_{i2}, the FOCs are given as follows:

$$\frac{d\pi_{A2}}{dx_{A2}} = \frac{x_{B2}}{(x_{A2}+x_{B2})^2}V_A - \frac{1}{(1+M)^\theta} + \frac{d\hat{\pi}_{A3}}{dx_{A2}} = 0; \tag{3.11}$$

$$\frac{d\pi_{B2}}{dx_{B2}} = \frac{x_{A2}}{(x_{A2}+x_{B2})^2}V_B - 1 + \frac{d\hat{\pi}_{B3}}{dx_{B2}} = 0. \tag{3.12}$$

Rewriting (3.11), we get

$$\frac{d\pi_{A2}}{dx_{A2}} = \frac{x_{B2}}{(x_{A2}+x_{B2})^2}V_A - 1 + \frac{d\hat{\pi}_{A3}}{dx_{A2}} + \left(1 - \frac{1}{(1+M)^\theta}\right) = 0. \tag{3.13}$$

Since there is a positive term in the LHS $\left(\frac{1}{(1+M)^\theta} < 1\right)$, the payoff function of A is maximised at a higher value of x_{A2}. Hence, in this case, the first period conflict investments of A increase because of third party intervention. Denoting the equilibrium levels of second period conflict in this case by x_{A2}^{TR}, x_{B2}^{TR}, and the aggregate level of conflict in the second period by $X_2^{TR} = x_{A2}^{TR} + x_{B2}^{TR}$, we get $X_2^{TR} > \hat{X}_2$.

Let us now turn to the effect of third party intervention in the second period, on the third period conflict investments for revenge. Let the third period conflict be denoted by $X_3^{TR} = x_{A3}^{TR} + x_{B3}^{TR}$. Since $\frac{dX_3^{TR}}{dx_{A2}} = \frac{R_A^2 R_B'}{(R_A+R_B)^2} > 0$ and since x_{A2} rises, hence due to third party intervention in the second period, total conflict in the third period increases. Let total conflict be $X_R^T = X_2^{TR} + X_3^{TR}$. Therefore, total conflict with third party intervention and with revenge is more than the total conflict without-third-party intervention and with revenge. That is, $X_R^T > \hat{X}$

To analyse more concrete cases, we look at the examples in Amegashie and Runkel (2012), but now augmented with the presence of the third party. The following proposition summarises the results in the case when the revenge functions, R's are symmetric across the players and so are the V's, the valuations of the resource for the players[11].

[11] A similar proposition under different parametric conditions ($(1+M)^\theta > 2$) is relegated to the Appendix (Appendix A.0.2) to avoid cluttering of the main body of the chapter.

Proposition 3.1. *Let $R_i(x_{j2}) = \alpha x_{j2}^{\phi}, i \neq j, V_A = V_B$ and $(1+M)^{\theta} < 2$.*

$$(i)\ \ X_R^T > X_T > \tilde{X},\ \ and\ \ X_R^T > \hat{X} > \tilde{X},\ \ when\ \ \phi < \frac{1}{2};$$
$$(ii)\ \ X_T > X_R^T > \hat{X} > \tilde{X},\ \ when\ \ 1 > \phi > \frac{1}{(1+M)^{\theta}};$$
$$(iii)\ \ X_T > X_R^T > \hat{X},\ \ and\ \ X_T > \tilde{X} > \hat{X},\ \ when\ \ \phi > 1.$$

Proof. Proof in Appendix A.0.2. ■

Observation 3.2. When ϕ lies between 1 and $\frac{1}{(1+M)^{\theta}}$ we get a complete relation between the conflict levels. The maximum conflict is when there is a third party and it is helping one of the players but no revenge and the minimum conflict is when there is conflict without both revenge and third party.

However, irrespective of whether or not there is revenge, conflict levels are now higher (more so if there isn't revenge) since $X_T > X_R^T > \hat{X} > \tilde{X}$.

Observation 3.3. In this case (case (*iii*)), as in a corresponding case of Amegashie and Runkel's (2012) Proposition 1, $\tilde{X} > \hat{X}$, i.e., the paradox of revenge is in effect. However with the introduction of the third party, we see that conflict invariably rises from that lower level.

Observation 3.4. The presence of a third party acting as an ally of one of the combatants accelerates the paradox of revenge. In other words, for a value of $\phi < 1$ the paradox of revenge is in effect ($X_T > X_R^T$) compared to without third party case. The presence of a third party as an ally of one of the combatants increases the conflict investment of the ally (and also the overall level of conflict) which increases the value effect of revenge of its opponent in the third period. Since, the elasticity of revenge is more than the cost-reduction factor of the ally, its opponent has a strong incentive to strike back thus reducing the second period effort of the ally.

Like in Amegashie and Runkel (2012), let us see how conflict is affected in the more asymmetric cases. The next proposition lays down the results when the valuation of revenge is different for the two players.

Proposition 3.2. *When $R_i(x_{j2}) = \alpha_i x_{j2}, \alpha_A \neq \alpha_B$ and $V_A = V_B$, then $X_T > X_R^T > \hat{X}$ and $X_T > \tilde{X} > \hat{X}$.*

Proof. Proof in Appendix A.0.3. ■

Observation 3.5. When third party intervenes as an ally of one of the parties, the level of conflict without revenge is more than with it, thus we observe paradox of revenge ($X_T > X_R^T > \hat{X}$). And also, in the presence of such third party intervention but in the absence of revenge, the level of conflict is higher than the original level ($X_T > \tilde{X}$).

Observation 3.6. $\tilde{X} > \hat{X}$ is just the paradox of revenge, as reported in Proposition 2 in Amegashie and Runkel (2012).

Proposition 3.3. *When* $R_i(x_{j2}) = x_{j2}, V_A \neq V_B$ *then* $X_T > X_R^T > \hat{X}$ *and* $X_T > \tilde{X} > \hat{X}$*.*

Proof. Proof in Appendix A.0.4. ■

Observation 3.7. With symmetric revenge functions but with asymmetric valuations, we have findings that are similar to that of Proposition 3.2.

Observation 3.8. This result is similar in spirit to Amegashie and Runkel's (2012) result in Proposition 3 where we see the paradox of revenge with the introduction of asymmetry in the valuations of the two players. i.e. $\tilde{X} > \hat{X}$.

Hence, from all the propositions in this section, the general message seems to be that the introduction of a third party that acts as an ally of one of the players is likely to lead to escalation of conflict, even overturning downward pressure on conflict that the self-deterrence effect of revengeful opponents might have.

Third Party Intervention as an Ally - SPNE

The literature on third party interventions, is divided, as to what the objective of third parties is, when they decide to intervene. Here we follow a formulation proposed in Chang, Potter and Sanders (2007), where the third party be a 'selfish-agent'. In their set-up the conflict between the parties is a territorial dispute and the third party maximises its expected payoff depending on who gets to keep hold of the land net of the military subsidies to its ally, given by

$$\pi_T = p_A K_A + p_B K_B - M, \tag{3.14}$$

where, $K_i, i = A, B$, is the strategic value of the resource to the third-party if combatant i wins the conflict which is given exogenously, $p_i, i = A, B$, is the probability that combatant i wins in the conflict, and M is the military subsidy provided to its ally. If the third-party doesn't intervene with the military subsidy then its expected payoff will remain, $\tilde{\pi}_T = p_A K_A + p_B K_B$.

We have already calculated the equilibrium conflict investments of the combatants when the third-party acts as an ally of the combatant A. So, without loss of generality let us assume $K_A > K_B \geq 0$, and hence the third party will be better-off if A wins the conflict and gets to keep the resource, so that the third-party would want to provide the military subsidy to A[12].

3.2.5 No revenge

Recall, that the equilibrium probabilities of winning of the combatants in the no revenge scenario are as follows:

$$p_{A2}^T = \frac{V_A(1+M)^\theta}{V_A(1+M)^\theta + V_B}, \tag{3.15}$$

$$p_{B2}^T = \frac{V_B}{V_A(1+M)^\theta + V_B}. \tag{3.16}$$

Thus, the probability of winning of combatant A (third party's ally) rises compared to the no-third-party-intervention case and the probability of winning of B falls compared to the no-third-party-intervention case. Putting the equilibrium values of the probabilities in the third-party's payoff function we get:

$$\pi_T = \left(\frac{V_A(1+M)^\theta K_A}{V_A(1+M)^\theta + V_B}\right) + \left(\frac{V_B K_B}{V_A(1+M)^\theta + V_B}\right) - M \tag{3.17}$$

Maximising the third-party's objective function in (4.33) w.r.t the military subsidy it provides to its ally, we get

$$\frac{d\pi_T}{dM} = \frac{V_A V_B \theta (1+M)^{\theta-1}(K_A - K_B)}{(V_A(1+M)^\theta + V_B)^2} - 1. \tag{3.18}$$

The third-party won't intervene as an ally of combatant A if its payoff is always falling with M, i.e, $\frac{d(\pi_T)}{dM} < 0$, i.e., the RHS of (3.18) is negative, i.e.,

$$(K_A - K_B) < \frac{(V_A(1+M)^\theta + V_B)^2}{V_A V_B \theta (1+M)^{\theta-1}} \tag{3.19}$$

$$\implies K_A < K_B + \frac{(V_A(1+M)^\theta + V_B)^2}{V_A V_B \theta (1+M)^{\theta-1}}. \tag{3.20}$$

[12] Chang, Potter and Sanders (2007) also used a similar argument to explain the third party's intervention in a conflict as an ally of one of the combatants.

Let $K_v = K_B + \frac{(V_A(1+M)^\theta + V_B)^2}{V_A V_B \theta (1+M)^{\theta-1}}$, be the reservation value of the resource for the third-party. Hence we conclude, that when the strategic-value of the resource for the third-party (when its ally wins) is less than the reservation value of K_v, then the third-party would choose to not intervene. Thus, the strategic-value of the resource for the third-party when its ally wins, must be sufficiently high for the third party to intervene.

3.2.6 With revenge

Now let us the consider this endogenous third-party intervention in the presence of revenge. After the conflict ends and the winner gets hold of the resource, the third-party can derive some strategic benefits out of the resource. But if the conflict continues out of revenge motivations then it is likely that it becomes difficult for the third-party to derive strategic benefits from the resource. Hence, we hypothesise the following payoff function of the third party:

$$\pi_T = p_A(K_A - R_A) + p_B(K_B - R_B) - M, \tag{3.21}$$

where, $R_i, i = A, B$ is the revenge function of i. The idea is even if i wins the conflict, the strategic value of the resource that accrues to the third party is lower by the amount of revenge investments. If the third-party doesn't intervene with the military subsidy then its expected payoff will remain, $\hat{\pi}_T = p_A(K_A - R_A) + p_B(K_B - R_B)$. Maximising (3.21) w.r.t M we get:

$$\frac{d\pi_T}{dM} = \frac{dp_A}{dM}(K_A - R_A) + \frac{dp_B}{dM}(K_B - R_B) - p_A\left(\frac{dR_A}{dM}\right) - p_B\left(\frac{dR_B}{dM}\right) - 1. \tag{3.22}$$

Simplifying (3.22) we get:

$$\frac{d\pi_T}{dM} = \frac{dp_A}{dM}(K_A - R_A - K_B + R_B) - p_A\left(\frac{dR_A}{dM}\right) - p_B\left(\frac{dR_B}{dM}\right) - 1. \tag{3.23}$$

The third party will not intervene if, $\frac{d\pi_T}{dM} < 0$. Simplifying (3.23) when $\frac{d\pi_T}{dM} < 0$, we get

$$(K_A - K_B) < \frac{1 + p_A(\frac{dR_A}{dM}) + p_B(\frac{dR_B}{dM})}{\frac{dp_A}{dM}} + (R_A - R_B) \tag{3.24}$$

$$\implies K_A < \frac{1 + p_A(\frac{dR_A}{dM}) + p_B(\frac{dR_B}{dM})}{\frac{dp_A}{dM}} + (R_A - R_B) + K_B \tag{3.25}$$

Now, $\frac{dR_A}{dM} = \frac{dR_A}{dx_{B2}}\frac{dx_{B2}}{dM} > 0,$ when, $\frac{dx_{B2}}{dM} > 0$ since $\frac{dR_A}{dx_{B2}} > 0$ (as per the formulation of the revenge function), conflict investment of B rises when a third-party intervenes as an ally of its opponent A when $V_B > V_A(1+M)^\theta$. As the conflict investment of A rises with the military subsidy, in order to win the confrontation the conflict investment of B also rises. Since, B values the resource more than that of A, its conflict investment increases with increase in A's conflict investment from the military subsidy provided by the third-party. Thus, revenge motivation of A rises with the military subsidy M. Similarly, $\frac{dR_B}{dM} = \frac{dR_B}{dx_{A2}}\frac{dx_{A2}}{dM} > 0$. Revenge motivation of B increases with the increase in military subsidy M.

3.2.6.1 Case 1: $V_B > V_A(1+M)^\theta$, i.e., $\frac{dx_{B2}}{dM} > 0$

Let $K_v' = \frac{1+p_A(\frac{dR_A}{dM})+p_B(\frac{dR_B}{dM})}{\frac{dp_A}{dM}} + (R_A - R_B) + K_B$ be the reservation-value of the resource in the presence of revenge. Now, it is evident $\frac{1+p_A(\frac{dR_A}{dM})+p_B(\frac{dR_B}{dM})}{\frac{dp_A}{dM}} + K_B > \frac{1}{\frac{dp_A}{dM}} + K_B$ and if the change in revenge of the two combatants is negligible then the reservation value of the resource for the third party increases in the 'with-revenge' case compared to 'no-revenge' case. Thus, $K_v' > K_v$. That is, the reservation value further increases. Hence the strategic value of the resource has to be even higher now for the third party to intervene as an ally of one of the combatants, when they are engaged in revenge of each other.

If $\left(p_A\left(\frac{dR_A}{dM}\right) + p_B\left(\frac{dR_B}{dM}\right)\right) < (R_B - R_A)\frac{dp_A}{dM}$ then $K_v' < K_v$, i.e the reservation value of the resource for the third-party when its ally A wins fall compared to the 'no-revenge' case reservation value. This fall can be explained as follows: if the revenge motivation of the ally's opponent, B, is very high compared to the ally, A, then the strategic value of the resource if B wins is very low for the third party, and thus the third party would want to help A more in order to get better strategic benefits from the resource.

3.2.6.2 Case 2: $V_B < V_A(1+M)^\theta$, i.e., $\frac{dx_{B2}}{dM} < 0$

In this case the results are interesting. Now, $\frac{dR_A}{dM} = \frac{dR_A}{dx_{B2}}\frac{dx_{B2}}{dM} < 0$, when, $\frac{dx_{B2}}{dM} < 0$ since $\frac{dR_A}{dx_{B2}} > 0$. $\frac{dx_{B2}}{dM} < 0$ when $V_B < V_A(1+M)^\theta$. The conflict investment of the opponent, B falls with increase in military subsidy M, which further implies the negative impact of military subsidy on the revenge factor of A, i.e revenge motivation of A falls with increase in M.

Here, the reservation value of the resource for the third-party depends on the impact of the military subsidy on the revenge motivation of the combatants.

If $(R_A > R_B)$ and if $\left(p_A\left(\frac{dR_A}{dM}\right) + p_B\left(\frac{dR_B}{dM}\right)\right) > 0$ then $K_v' > K_v$, the reservation value for the third party rises compared to the 'no-revenge' case. This can be explained in the following way: when the revenge motivation of A is comparatively more than that of B it puts a negative impact on the strategic-value of the resource for the third-party when its ally A wins. And when the positive impact of military subsidy provided by the third-party on the revenge function of B is more than the negative impact of it on revenge function of A then the strategic-benefits from the resource if A wins falls for the third-party in a conflict with revenge. Which implies that only when the reservation-value is very high nullifying this negative impacts from the intervention then only the third-party will intervene in a conflict with revenge.

If $(R_A < R_B)$ and if $\left(p_A\left(\frac{dR_A}{dM}\right) + p_B\left(\frac{dR_B}{dM}\right)\right) < 0$ then $K_v' < K_v$, the reservation value for the third-party falls compared to the 'no-revenge' case. This can be explained in the following way: when the revenge motivation of B is comparatively more than that of A, the third-party would want to help its ally A more. And also when the negative impact of military subsidy on revenge function of A is more than the positive impact of it on the revenge function of B, then the strategic-benefits from the resource if A wins, rises for the third party in a conflict with revenge. This shows that when the negative-impact of the third party intervention is more on the revenge-factor, then the third-party would want to help its ally more, thus reducing the reservation value of the resource for the third party.

In short, whether or not a third party decides to intervene as an ally or not is an endogenous choice on the part of the third party and it importantly depends on whether or not the conflictual parties harbour revenge motivations against each other or not.

3.2.7 Comparing the payoffs of the players

In this section we briefly discuss about the payoffs of A, B in the above discussed cases. In the benchmark case of no revenge and no third party intervention, the payoffs are as follows:

$$\tilde{\pi}_i = \frac{V_i^3}{(V_A + V_B)^2}, \text{where}, i = A, B. \tag{3.26}$$

When there is no revenge but there is third party intervention as an ally of A, the payoffs are as follows:

$$\pi_A^T = \frac{V_A^3(1+M)^{2\theta}}{(V_A(1+M)^\theta + V_B)^2}, \tag{3.27}$$

$$\pi_B^T = \frac{V_B^3}{(V_A(1+M)^\theta + V_B)^2}. \tag{3.28}$$

Comparing we get the following:

$$\tilde{\pi}_A < \pi_A^T.$$
$$\tilde{\pi}_B > \pi_B^T.$$

As expected, we find that sans revenge, the payoff of A, the player who gets help from a third party ally, increases when the third party acts as its ally relative to when it does not get help from the third party. However, the payoff of player B decreases when there is third party intervention in the form of an ally of its opponent compared to the no-third-party intervention case (sans revenge in both situations). Intuitively, this increase in payoff of combatant A can be explained by the fact that it attains an advantageous position in the conflict since it has the support of an ally in the conflict.

Now, let us compare the payoffs of the cases where there is no third party but revengeful intentions may or may not be there. To keep it simple we won't compare the payoffs taking into consideration all the three revenge functions mentioned in Propositions 3.1, 3.2 and 3.3, but use a complete symmetric case to show the change in payoffs of the conflictual players in the presence of revenge. The payoffs of A and B in the presence of revenge (but no third party intervention) are as follows:

$$\hat{\pi}_i = \frac{x_{i2}}{(x_{A2} + x_{B2})} V_i - x_{i2} + \hat{\pi}_{i3}, \text{where}, i = A, B.$$

Comparing these with the payoffs in the benchmark case of no revenge and no third party intervention as given in (3.26), we get:

$$\tilde{\pi}_A - \hat{\pi}_A = V_A\left(\frac{\tilde{x}_{A2}}{(\tilde{x}_{A2} + \tilde{x}_{B2})} - \frac{\hat{x}_{A2}}{(\hat{x}_{A2} + \hat{x}_{B2})}\right) + (\hat{x}_{A2} - \tilde{x}_{A2}) - \frac{R_A^3}{(R_A + R_B)^2}. \tag{3.29}$$

Similarly, for player B. Since, we are considering the symmetric case where the valuation of the resource for both the combatants is equal, i.e., $V_A = V_B$, we can find that the equilibrium conflict investments of both the players are also equal. Thus, $\tilde{x}_{A2} = \tilde{x}_{B2}$ and $\hat{x}_{A2} = \hat{x}_{B2}$. The first part of RHS of (3.29) becomes zero, hence equation (3.29) can be written as:

$$\tilde{\pi}_{A2} - \hat{\pi}_{A2} = 0 - (\tilde{x}_{A2} - \hat{x}_{A2}) - \frac{R_A^3}{(R_A + R_B)^2}. \tag{3.30}$$

Now, $\tilde{x}_{A2} > \hat{x}_{A2}$ because of the self-deterrence effect the second period conflict investment of the conflictual players in the presence of revenge. Thus, equation (3.30) is negative. Hence, we get:

$$\hat{\pi}_A > \tilde{\pi}_A.$$

Similarly, for combatant B. Thus, the presence of revenge increases the payoff of the conflictual players in a completely symmetric setting which also explains why the conflicting parties want to exact revenge in the first place - the presence of revenge increases the payoff of the players engaged in conflict. The presence of a third party as an ally of one of the players (here, A) in the presence of revenge, further increases the payoff of the combatant-ally (here, A), i.e. $\pi_A^{TR} > \pi_A^T$. The payoff of the other combatant (here, B) is ambiguous (rises because of revenge, falls because of third-party intervention), but since revenge by one of the players implies revenge for both in the subgame, we will have revenge being taken by both.

However, if we compare the payoffs of the third party in the presence of revenge and no revenge, then the payoff of the third-party falls in the presence of revenge because revenge motivations of the players enters as a negative component in the payoff function of the third-party. The more the revenge motivation of the players, lesser the strategic benefits the third-party can derive from the resource. In short, $\pi_T^T > \pi_T^{TR}$.

In summary we have, the payoffs of the conflictual players increase in the presence of revenge (or at least it increases for one of them, the ally). Thus, the dominant strategy for the conflictual players in the 'revenge' subgame would be to play 'revenge'. Given this, however, the decision of the third-party to intervene in the conflict as an ally would depend on the strategic value of the resource ($K_i, i = A, B$). Listed below are the SPNE:

1) When the strategic value of the resource if A wins is sufficiently high, i.e., $K_A > \frac{1+p_A(\frac{dR_A}{dM})+p_B(\frac{dR_B}{dM})}{\frac{dp_A}{dM}} + (R_A - R_B) + K_B$, then third party will intervene in any SPNE. Players A, B will choose $(x_{A2}^{TR}, x_{B2}^{TR})$ in the subgame following third party intervening as an ally of A, while they choose $(\hat{x}_{A2}, \hat{x}_{B2})$ in the subgame following no intervention by the third party. Moreover, both players A and B exact revenge on each other, irrespective of third party intervention. The bold blue lines in Figure 3.2 depicts the SPNE.

2) When the strategic value of the resource if A wins is not sufficiently high, such that, $K_A < \frac{1+p_A(\frac{dR_A}{dM})+p_B(\frac{dR_B}{dM})}{\frac{dp_A}{dM}} + (R_A - R_B) + K_B$, then third party will not intervene in any

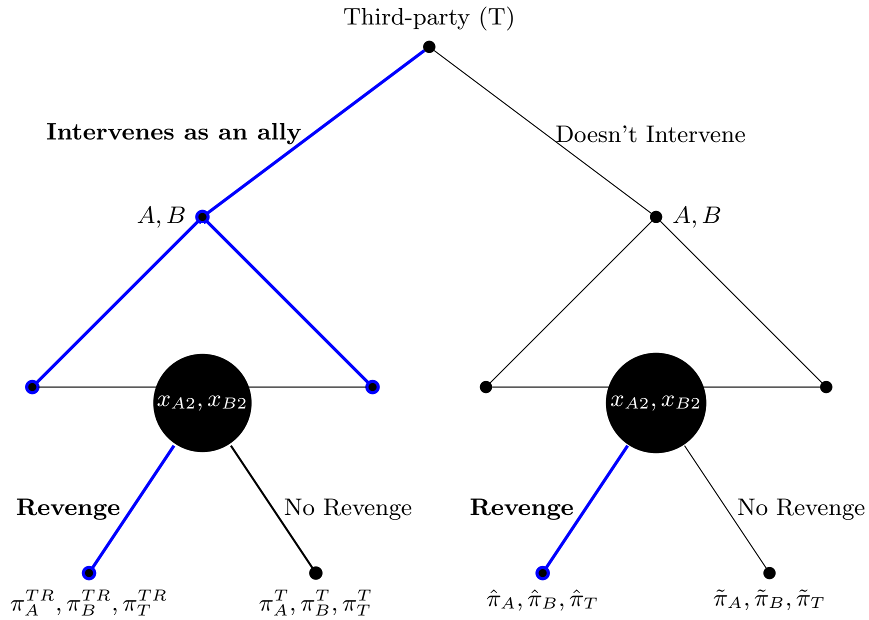

FIGURE 3.2: SPNE when third party intervenes as an ally and there is revenge.

SPNE. However, choices of players A, B in the conflict and revenge subgames, remain the same, i.e., they will choose $(x_{A2}^{TR}, x_{B2}^{TR})$ in the subgame following third party intervening as an ally of A, they choose $(\hat{x}_{A2}, \hat{x}_{B2})$ in the subgame following no intervention by the third party. And they exact revenge on each other thereafter.

Empirically we can also deduce the endogeneity of third party intervention (as an ally), depending on whether or not there is revenge motivation on the part of the conflictual parties. See, for example, Iraq's role in Israel-Palestine conflict in Section 3.4.4 below.

3.3 Third Party Intervention as an 'Idealist'

In this section, our model is augmented by a third party where the third party is now an 'idealist' and therefore tries to reduce the overall level of conflict. This is in keeping with a large part of literature on third party intervention where the natural assumption is that third parties intervene on humanitarian grounds to minimise loss of lives and property in conflicting countries (see Regan (1998) and Siqueira (2003), for example), and hence the overall intention is to reduce conflict levels. The third party takes a neutral position and uses transfers $M, M \geq 0$ to increase the cost of conflict of both the players. Let $P(M)$ be the cost-increase function, such that $P'(M) > 0, P''(M) < 0$ and $P(M) = 1$ when $M = 0$, i.e., $P(M) \geq 1$. This formulation was used by Chang and

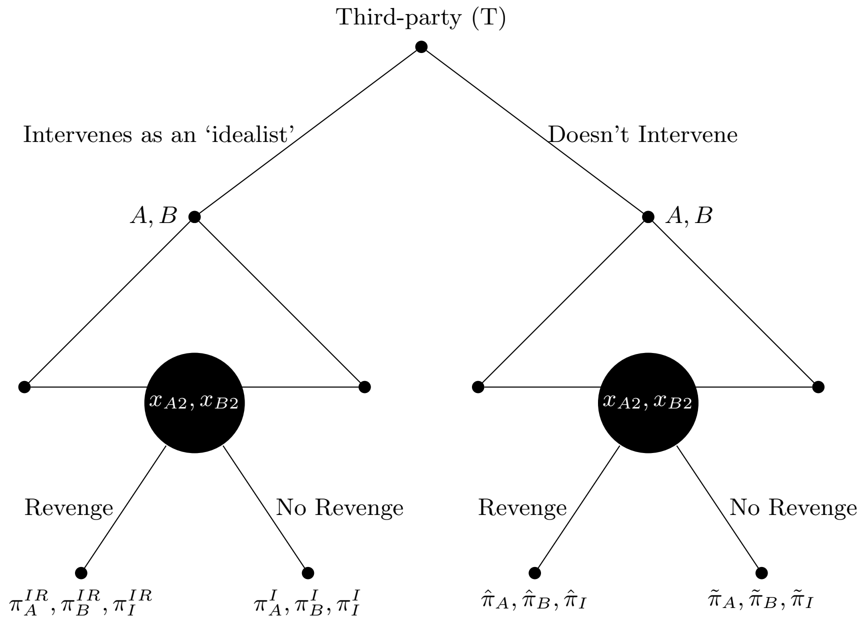

FIGURE 3.3: Third-party intervening (or not) as an 'idealist' in the presence (or absence) of revenge motivations

Sanders (2008). Also refer to an idealistic intervention by the UN Security Council in the Eritrean-Ethiopian conflict, as given in Section 3.4.5 below.

The nature of intervention is now different, otherwise it is re-presentable in the same game tree as depicted in Figure 3.1. Figure 3.3 represents the sequential game with different strategies of the third party. Notice that the different possibilities in this case remain the same as in the game depicted in Figure 3.1 though the analysis will be different given the specific nature of intervention. For clear exposition though, we enumerate the different paths once again here. Like before, the payoffs at the end notes are listed as (payoff of A, payoff of B, payoff of T) respectively.

(i) The third party doesn't intervene and there is no revenge $(\tilde{\pi}_A, \tilde{\pi}_B, \tilde{\pi}_I)$; (See section 3.3.1 below)

(ii) The third party doesn't intervene and A, B exact revenge $(\hat{\pi}_A, \hat{\pi}_B, \hat{\pi}_I)$; (See section 3.3.2 below)

(iii) The third party intervenes but there is no revenge $(\pi_A^I, \pi_B^I, \pi_I^I)$; (See section 3.3.3 below)

(iv) The third party intervenes and there is revenge $(\pi_A^{IR}, \pi_B^{IR}, \pi_I^{IR})$; (See section 3.3.4 below)

Like before, we use backward induction to find the SPNE of the game. We begin by looking at the Nash Equilibria of the subgames following third party intervention (as an idealist) in this case.

Subgame Equilibrium - Conflict and Revenge

The different possible subgames and the analysis therein are as follows:

3.3.1 No third party intervention and no revenge

This is the same benchmark case of no third party intervention and no revenge as given in Section 3.2.1.

3.3.2 No third party intervention but revenge

This is the same case of no third party intervention but the presence of revenge as given in Section 3.2.2.

3.3.3 With third party intervention as an idealist and without revenge:

Since there is no revenge there is no third period conflict. The payoff functions of players A and B are:

$$\pi_{i2} = P_{i2}V_i - P(M)x_{i2}, i = A, B. \tag{3.31}$$

Maximising w.r.t x_{i2} simultaneously and assuming interior solutions we get the FOCs as:

$$\frac{d\pi_{i2}}{dx_{i2}} = \frac{x_{j2}}{(x_{A2} + x_{B2})^2}V_i - P(M) = 0, i \neq j. \tag{3.32}$$

The equilibrium levels of period 2 conflict investments, x^I_{A2}, x^I_{B2}, and the aggregate level of conflict $X_I = x^I_{A2} + x^I_{B2}$ are as follows:

$$x^I_{i2} = \frac{V_i^2 V_j}{P(M)(V_A + V_B)^2}; \quad i, j = A, B, i \neq j; \tag{3.33}$$

$$X_I = \frac{V_A V_B}{(P(M))(V_A + V_B)}. \tag{3.34}$$

Observation 3.9. Comparing $\tilde{X}, X_T$ and X_I, it is clear that $X_T > \tilde{X} > X_I$. The idealistic intervention by the third party reduces the aggregate level of conflict, since the third party increases the cost of conflict for both the players.

3.3.4 With third party intervention as an 'idealist' and with revenge

Proceeding, in the same way as before, the second period payoff functions of the players will be as follows:

$$\pi_{A2} = \frac{x_{A2}}{x_{A2}+x_{B2}}V_A - P(M)x_{A2} + \hat{\pi}_{A3}; \tag{3.35}$$

$$\pi_{B2} = \frac{x_{B2}}{x_{A2}+x_{B2}}V_B - P(M)x_{B2} + \hat{\pi}_{B3}. \tag{3.36}$$

Maximising the payoff functions w.r.t x_{i2}, the FOCs are as follows:

$$\frac{d\pi_{A2}}{dx_{A2}} = \frac{x_{B2}}{(x_{A2}+x_{B2})^2}V_A - P(M) + \frac{d\hat{\pi}_{A3}}{dx_{A2}} = 0; \tag{3.37}$$

$$\frac{d\pi_{B2}}{dx_{B2}} = \frac{x_{A2}}{(x_{A2}+x_{B2})^2}V_B - P(M) + \frac{d\hat{\pi}_{B3}}{dx_{B2}} = 0. \tag{3.38}$$

From the first order conditions it is evident that the point of maximisation of the payoff functions is less than that of the FOCs with revenge and without third party intervention case (the Amegashie and Runkel model, 2012). The second period conflict therefore falls additionally (other than the self-deterrence effect leading to the paradox of revenge) because of the idealistic intervention.

Let the equilibrium levels of conflict investments be x_{A2}^{IR} and x_{B2}^{IR}, the aggregate conflict in the second period be X_2^{IR}, that in the third period be X_3^{IR}, and the total aggregate level of conflict be X_R^I. Since conflict investments of both A and B fall in the second period, conflict investments in the revenge stage (in the third period) also fall. Therefore, total conflict falls because of the third party idealistic intervention in this case, i.e., $\hat{X} > X_R^I$.

Proposition 3.4. *Let $R_i(x_{j2}) = \alpha x_{j2}^{\phi}, i \neq j, V_A = V_B$. Then we have the following:*

(i) $\hat{X} > X_R^I > X_I$, *and* $\hat{X} > \tilde{X} > X_I$ *when* $\phi < 1$;

(ii) $\tilde{X} > \hat{X} > X_R^I > X_I$ *when* $1 < \phi < P(M)$;

(iii) $\tilde{X} > X_I > X_R^I$ *and* $\tilde{X} > \hat{X} > X_R^I$ *when* $\phi > P(M)$.

Proof. Proof in Appendix A.0.5. ■

Observation 3.10. The most interesting cases seem to when $\tilde{X} > \hat{X} > X_R^I$. This corresponds to the 'paradox of revenge' in effect, but it falls much more with idealistic mediation. And this seems to be the case when $\phi > 1$. Recall that ϕ is the elasticity of the benefit of revenge of one player with respect to the conflict investment of the opponent in the previous period. Hence idealistic efforts by a third party are more fruitful in deterring revengeful opponents in reducing conflict, the more elastic their revenge functions are.

Observation 3.11. It is also interesting to note that when $\phi < 1$, $\hat{X} > \tilde{X} > X_I$, that is even when the paradox of revenge does not work, only such idealistic mediation may reduce conflict.

Observation 3.12. The presence of a third party acting as an idealist in a conflict decelerates the paradox of revenge. In other words, for a value of $\phi > P(M) > 1$, the paradox of revenge is in effect whereas in the without-third-party case the paradox of revenge is in effect when $\phi > 1$. Note that the conflict investments of both the combatants fall due to increase in cost of conflict in the second period thus leading to fall in the value-effect of revenge of both the combatants and also fall in the overall level of conflict. Hence, the paradox will be in effect only when the elasticity of the benefit of revenge (ϕ) is very high i.e. more than the cost-increase function ($P(M)$) in which case, the incentive to strike back is high for both the combatants thus reducing the conflict efforts of both the combatants initially.

Proposition 3.5. *Let $R_i(x_{j2}) = \alpha_i x_{j2}, \alpha_A \neq \alpha_B$ and $V_A = V_B$. Then $\tilde{X} > X_I > X_R^I$ and $\tilde{X} > \hat{X} > X_R^I$.*

Proof. Similar to proof of Proposition (3.2) in the Appendix. ■

Observation 3.13. In a similar vein to that of observation 3.10, $\tilde{X} > \hat{X} > X_R^I$ which is now true even when valuations are similar though revenge functions are asymmetric. That is, idealistic mediation reduces conflict more than that in the reduction that happens as a result of paradox of revenge.

Observation 3.14. In fact, since $\tilde{X} > X_I$, we know that even in the absence of revenge motivations, idealistic mediation reduces conflict. However, since $X_I > X_R^I$, we know that conflict falls more with revenge than without, under idealistic intervention.

Proposition 3.6. *When $R_i(x_{j2}) = x_{j2}, V_A \neq V_B$ then $\tilde{X} > X_I > X_R^I$ and $\tilde{X} > \hat{X} > X_R^I$.*

Proof. Similar to proof of Proposition (3.3) in the Appendix. ■

In this case the revenge functions are symmetric but the initial valuation for the resource is different. However, the findings with respect to the conflict levels are exactly similar to that in Proposition 3.5.

Third Party Intervention as an 'Idealist'- SPNE

In this case, when the third party intervenes as an 'idealist', its goal is to reduce the level of conflict (Siqueira, 2003). Hence, its payoff function is given by:

$$\pi_I = -X - M, \tag{3.39}$$

where, X is the total level of conflict, and M is the transfers it uses to increase the cost of conflict for the conflictual players. If the third-party doesn't intervene then its payoff remains equal to the negative of the total conflict level. Hence, without revenge, its payoff is: $\tilde{\pi}_I = -\tilde{X}$, while with revenge, its payoff is: $\hat{\pi}_I = -\hat{X}$.

Starting with the no-revenge scenario, in (3.34), we saw that the total level of conflict falls because of the intervention, thus having a positive impact on the third-party's payoff. However, the third-party will not intervene if its payoff keeps falling with its transfers, i.e., $\frac{d\pi_I}{dM} < 0$.

$$\frac{d\pi_I}{dM} = -\frac{dX_I}{dM} - 1 < 0. \tag{3.40}$$

Putting X_I from (3.34) in (3.40), we get,

$$\frac{d\pi_I}{dM} = \frac{V_A V_B P'(M)}{(P(M))^2 (V_A + V_B)} - 1 < 0. \tag{3.41}$$

Simplifying (3.41) we get,

$$\frac{V_A V_B}{(V_A + V_B)} = \tilde{X} < \frac{(P(M))^2}{P'(M)} \tag{3.42}$$

From (3.42) we can see that the LHS of the equation is the conflict level with no-revenge-no-third-party, $\tilde{X}$. Thus, from (3.42) it is clear that the third-party will only intervene with conflict-reducing idealistic motive when the level of conflict without its intervention is sufficiently high and its intervention with transfers can have a significant impact to reduce the level of conflict.

The presence of revenge enters as a negative component in the third-party's payoff function because of the additional conflict taking place in the third-period out of revenge. However when the motive of the third-party is to reduce the level of the conflict and if the presence of revenge reduces the level of conflict when there is more pronounced self-deterrence effect than value effect, then the presence of revenge can have a positive impact on the third-party's payoff. Thus, if there is paradox of revenge operating, then $\pi_I^{IR} > \pi_I^I$ (with or without intervention). But if there is no paradox of revenge then $\pi_I^{IR} < \pi_I^I$ (with or without intervention).

When the third-party intervenes with a conflict-reducing motive and without any revenge the payoffs of the combatants are as follows:

$$\pi_A^I = \left(\frac{V_A}{(V_A + V_B)}\right) V_A - P(M) \left(\frac{V_A^2 V_B}{P(M)(V_A + V_B)^2}\right)$$

$$\implies \pi_A^I = \frac{V_A^3}{(V_A + V_B)^2} = \tilde{\pi}_A \tag{3.43}$$

Similarly, the payoff of combatant B can be checked. From, equation (3.43) it is evident that the payoff of the combatants in presence of a third-party intervention as an idealist and no revenge is equal to the payoff of the combatants without any third-party intervention and no revenge as shown in equation (3.26). These equal payoffs of the combatants can be explained from the fact that when a third-party intervenes as an idealist it increases the cost of conflict for both the combatants thus reducing their conflict-investments, and since the conflict-investments of both the combatants fall their probability of winning remains the same and the increase in the cost factor is nullified by the fall in the conflict-investment factor. None of the combatants gain a favourable position in the conflict with respect to the other. Thus, this idealistic intervention only reduces the conflict-investments of the the combatants leaving their payoffs unchanged. Thus; $\pi_A^I = \tilde{\pi}_A$ and $\pi_B^I = \tilde{\pi}_B$. A similar explanation holds for the with third-party intervention (as an idealist) and with revenge scenario. Hence, when the third-party intervenes as an idealist, the payoffs of the combatants will be higher in the presence of revenge. The SPNE of the whole game are as follows:

1) The third-party will intervene as an 'idealist' when its intervention can have a significant negative impact on the level of conflict (that is reduce the level of conflict), i.e., $-\frac{dX}{dP(M)} \geq \frac{1}{P'(M)}$ where $X = X_2 + X_3$ total level of conflict and $\frac{dX}{dP(M)} < 0$. Thus, the SPNE will be: the third party intervenes as an idealist; Players A, B will choose $(x_{A2}^{IR}, x_{B2}^{IR})$ in the subgame following third party intervening as an idealist, while they choose $(\hat{x}_{A2}, \hat{x}_{B2})$ in the subgame following no intervention by the third party. Moreover, both players A and B exact revenge on each other, irrespective of third party intervention. The bold blue lines in Figure 3.4 depicts the SPNE.

2) Third party won't intervene as an 'idealist' when its intervention with the transfers won't have a significant impact on the level of conflict (reduce the level of conflict), i.e., $-\frac{dX}{dP(M)} < \frac{1}{P'(M)}$ where $X = X_2 + X_3$ total level of conflict and $\frac{dX}{dP(M)} < 0$. Thus, the SPNE will be: the third party does not intervene as an idealist; Players A, B will choose $(x_{A2}^{IR}, x_{B2}^{IR})$ in the subgame following third party intervening as an idealist, while they choose $(\hat{x}_{A2}, \hat{x}_{B2})$ in the subgame following no intervention by the third party. Moreover, both players A and B exact revenge on each other, irrespective of third party intervention.

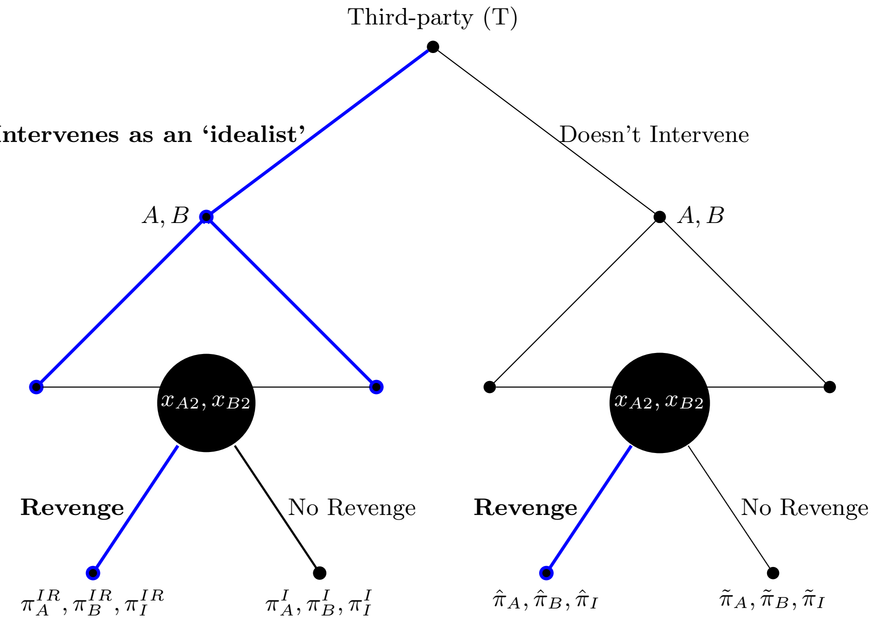

FIGURE 3.4: SPNE when third party intervenes as an idealist and there is revenge.

3.4 Narrative Evidence

In this section we specifically look at real-life examples of conflicts where the two potent influences of revenge and third party intervention, are ubiquitous. Specifically, we try to highlight the specific SPNE that best fits the real life conflict scenario that we cite here[13].

3.4.1 Conflict in Yemen

Yemen has been reeling under conflicts for decades and faces one of the worst humanitarian crises (see Munshi (2020) for many other details of this discussion[14]). Third parties play very important roles in the politics in general, and the conflicts in particular, in Yemen.

The Houthis belong to the Zaydi sect of Shias. The Zaydis had ruled over Northern Yemen for almost a millennium before being overthrown in a coup in 1962. For next three decades, the Zaydis were marginalised both politically and economically by the

[13]However, there are two SPNEs (out of four) where the third party doesn't intervene and the conflictual parties engage in revenge. It's hard to find evidence for something that doesn't happen, simply because it doesn't happen. So most of our empirical evidence points out instances of something happening (third party intervening, in this case). Though for the case of Israel-Palestine conflict (see Section 3.4.4), the role of Iraq discernibly changes from being an ally to not being an ally, thereby bringing out the case of 'no intervention' from 'intervention' by a third party.

[14]Also see "'Dancing on the Heads of Snakes": A Glimpse into Yemen' by Soumyanetra Munshi (December 24 2020) for a general account of the history of the Yemeni conflict.

government. Finally, Mohammad Badr al-Din Houthi started the Ansar Allah movement which carried out extensive military campaigns during 2002-09 in the hope of securing greater political participation. Hence we also see long, protracted, revengeful warfare that has been ongoing for decades.

The Houthis seized the presidential palace in January 2015, leading President Mansour Hadi and his government to resign. Beginning in March 2015, a coalition of Gulf states led by Saudi Arabia launched a campaign of economic isolation and air strikes against the Houthi insurgents, with U.S. logistical and intelligence support.

The conflict in Yemen best fits the SPNE where the third party intervenes as an ally and there is violent revengeful interactions among the conflicting players (see Figure 3.2). Saudi-Arabia has stayed as an ally of the former Hadi government and carried out many air-strikes against the Houthis.

In fact, recall that the parametric restriction for such an equilibrium to exist, was essentially a high strategic value of the resource to the third party, should the ally win. In fact, the strategic importance of a Sunni-dominated Hadi government would be unquestionably immense to Saudi Arabia which sees itself as a leading Sunni Muslim power.

3.4.2 Conflict in Ireland

Here we consider the conflict between Catholics and Protestants in North Ireland[15]. When Ireland was under the control of England, large numbers of English and Scottish people were encouraged to settle in the north of Ireland. While most of the native Irish were Catholic, most of the settlers were Protestant. Even after the Irish independence in 1921, the struggle continued to get Ulster back from the British. The biggest obstacle was that the Protestants, who were happy as citizens of the United Kingdom did not want to be liberated.

In this conflict that the British army was more of an ally of the Protestants and their bias against the Catholics lead to escalation of the conflict. But the majority on both sides were tired of all the violence and the personal losses caused by it. As a result many saw the Good Friday Peace Agreement in 1998 as a milestone for peace, since it was signed by the most important political leaders on both sides.[16]

[15] Note that this example was also cited by Amegashie and Runkel (2012) as a good example of demonstration of the 'paradox of revenge'. However, they admit that "Of course, there may be other reasons for the resolution of the conflict (e.g., third-party intervention, faction asymmetries), but the self-deterrence effect identified in our analysis may also contribute to the explanation of this observation..." (Pg. 315) Hence our analysis can be thought of complementing Amegashie and Runkel's (2012) observation, by incorporating third parties.

[16] https://www.britannica.com/event/The-Troubles-Northern-Ireland-history

This conflict also best fits the SPNE where the third party (Britain) acts as an ally (of the Protestants) thereby intensifying existing conflict situation (see Figure 3.2). Again, corroborating the parametric restriction required for this case, the significant strategic importance of the Protestants in North-Ireland (compared to Catholics) to the Britain, was quite unquestionable.

3.4.3 The 2019-2020 Persian Gulf crisis

This crisis started in May 2018 when United States withdrew from the Joint Comprehensive Plan of Action (JCOPA) nuclear deal, which reinstated the sanctions against Iran. As a result of the sanctions Iran's economy faced a sharp downturn. Military tensions between Iran and the United States escalated in 2019 amid a series of confrontations involving the US, Iran, and Saudi Arabia. In this conflict Saudi Arabia supported US. Iran and Saudi Arabia has been loggerheads since the Iranian Revolution in 1979. The rivalry today is primarily a political and economic struggle exacerbated by religious differences. Iran is largely a Shia Muslim and Saudi Arabia sees itself as the leading Sunni Muslim power. On 31 December 2019, Iran-backed militiamen attacked the outer perimeter of the U.S. Embassy in Baghdad, prompting American diplomats to evacuate to safe rooms. US retaliated, on 3rd January 2020 President Donald Trump approved the targeted killing of Iranian Major General Qasem Soleimani in Baghdad. Iran's supreme leader Ali Khamenei pledged to exact revenge on US.[17]

The 2019-2020 Persian Gulf Crisis is also in line with the SPNE where the third party (Saudi Arabia) intervenes as an ally (of US) leading to worsening of existing conflictual scenario which is also revengeful (Figure 3.2).

3.4.4 Israel-Palestine conflict

Israelis and Palestinians have clashed over claims to the Holy Land for decades, a conflict that has long been one of the world's most intractable.

Iraq has played a crucial role as a third party in this conflict. In 1948 when the State of Israel declared its existence, Iraq (with other Arab countries) immediately declared war on Israel, in order to restore Arab control on the whole of Palestine. Iraq has been implacably hostile to Israel and has always supported Palestine (has acted as an ally of Palestine).

[17] 1)https://en.wikipedia.org/wiki/2019%E2%80%932020_Persian_Gulf_crisis
2)https://en.wikipedia.org/wiki/Iran%E2%80%93Saudi_Arabia_proxy_conflict

But as time passed and the Israel-Palestine conflict escalated with revengeful attacks on each other, there has been a perceivable change in Iraq's role in the conflict. In fact, in 2019, Iraqi Foreign Minister, Mohammed Ali al Hakim, said that it would be Iraqi policy to support a two-state solution between the Israelis and the Palestinians[18]. The whole shift of role of the Arab and Muslim countries of the world towards Israel from hostility to normalisation shows how the strategic importance (of the ally, Palestine) and revengeful intentions of conflictual players can influence a third party's decision to intervene (as an ally in this case).

The evolved role of Iraq in the Israel-Palestine conflict seem to best fit the SPNE where the third party does not intervene and there is revengeful interaction among the conflicting players. As laid down in the parametric restriction for such an SPNE to exist, the relative strategic importance of Palestine seem to be low enough for Iraq to intervene as an ally of Palestine.

3.4.5 Eritrean - Ethiopian War

The Eritrean - Ethiopian War also known as Badme war was a conflict that took place between Ethiopia and Eritrea from May 1998 to June 2000, with final peace agreed to in 2018.

According to a ruling by an international commission in The Hague, Eritrea broke international law and triggered the war by invading Ethiopia. After the war ended, the Eritrea - Ethiopia Boundary Commission, a body founded by the UN, established that Badme, the disputed territory at the heart of the conflict, belongs to Eritrea. On 5 June 2018, the Ethiopian People's Revolutionary Democratic Front, headed by Prime Minister Abiy Ahmed, agreed to fully implement the peace treaty signed with Eritrea in 2000, with peace declared by both parties in July 2018[19].

The United Nations Security Council intervened as an idealist in this conflict. In May 2000 it established a committee to monitor and implement an arms embargo against Ethiopia and Eritrea in response to continuing hostilities between the two countries over the borders[20]. This conflict best fits our SPNE where the third-party intervenes as an idealist in a conflict of revengeful interactions and is thus able to reduce conflict levels significantly (see Figure 3.4).

[18] https://www.trtworld.com/opinion/why-iraq-is-starting-to-normalise-ties-with-israel-23676 (accessed 18/12/2021)

[19] https://en.wikipedia.org/wiki/Eritrean-Ethiopian-War (accessed 19/12/2021)

[20] https://www.un.org/securitycouncil/content/repertoire/sanctions-and-other-committees (accessed 10/6/2021)

3.5 Conclusion

The literature on conflict has looked at revenge and at third party interventions, but mostly separately. Yet, most conflicts in the real world are characterised by significant presence of both. This chapter attempts to explore implications that both of them acting together might have on a conflict situation. In fact, it also attempts to endogenize the decision of the third party whether or not to intervene in the first place (though the capacity in which it may intervene, as an ally of one of the combatants or an idealist aiming to reduce overall conflict, is exogenously considered).

This chapter analyses how the presence of a third-party in a conflict can influence the level of conflict with or without revenge. An intervention as an ally of one of the conflicting parties, increases the overall conflict levels, with and without revenge motivations, while an intervention as an 'idealist' decreases the overall levels of conflict, with and without revenge motivations. We find that the combatants are better off exacting revenge on each other, irrespective of whether the third party has intervened or not (either as an ally of one of them or as an 'idealist'). Given this, the third party intervenes as an ally only when the (strategic) value of the resource, if its ally wins, is sufficiently high. While the third party intervenes as an idealist only if its intervention can significantly reduce resultant conflict levels.

Chapter 4

Conflict under Incapacitation and Revenge: A Game-Theoretic Exploration

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This paper won the 8th Walter Isard Annual Award for the Best Article (2024) in *Peace Economics Peace Science and Public Policy*[1].

4.1 Introduction

Imagine an ongoing conflict between two parties, where parties harbour revenge motivations. Any such conflict can be conceptually characterised by three effects: First, an *incapacitation effect*, that is violence by one side is likely to inflict damage on the resources of the other so that the ability of the opponent to react may be compromised (see Jaeger and Paserman (2008) for example); second, a *value effect or vengeance effect* of revenge, that is violence inflicted by the opponent can provoke a party to engage in revengeful violence since that leads to greater satisfaction to the sufferer in the first place; third, a *self-deterrence effect*, that is a party can be deterred from using violence in apprehension that it will cause further retaliatory violence from its opponent (see Amegashie and Runkel (2012), for the last two effects). Incapacitation and self-deterrence effects of revenge are likely to reduce violence (since it is likely to reduce

[1]https://www.degruyterbrill.com/document/doi/10.1515/peps-2025-0086/html

the ability to retaliate for a given level of violence inflicted by the opponent as well as lead to fear of retaliation by the opponent) whereas the value effect of revenge is likely to increase violence (since exacting revenge is likely to yield greater satisfaction to the sufferer of violence). Hence, how overall violence level varies when all these effects are at play, is not a priori obvious. This chapter attempts to shed some light on this question by studying a simple game-theoretic model.

Revengeful behaviour has been found in plenty of human societies[2]. For example, revenge and honor play a crucial role in Pashtun communities for maintaining order and conformity (Muhammed et al. 2016). Revenge is believed to be one of major cause of continuing conflict other than the real cause of conflict. As described by Amegashie and Runkel (2012), the value-effect of revenge captures the conflict-enhancing influence of revenge where a party derives satisfaction by engaging in inflicting further violence on an opponent in response to an initial violent attack by the opponent. Thus, the presence of revenge can exacerbate the conflict[3]. However, the presence of revenge can also lead to de-escalation of the conflict through the 'self-deterrence effect', which essentially means that a party may engage in less violence in fear of future retaliation by its opponent. And if the self-deterrence effect is stronger than the value effect of revenge, then the presence of revenge can lead to de-escalation of the conflict. Amegashie and Runkel (2012) calls de-escalation of conflict in the presence of revenge as 'paradox of revenge'.

In short, the presence of revenge can lead to both conflict escalation or de-escalation depending on which effect is more pronounced, the value effect of revenge or self-deterrence effect of revenge. However, in reality, a conflicting party will engage into further conflict out of revenge only when it has capability to do so. In other words, it is possible that the violence inflicted by its opponent has been so incapacitating that no matter how much it wants to exact revenge, it is either unable to do it or able to do it in much smaller amount. Let us consider the (ongoing) Israel-Palestine conflict; Israel has always been able to strongly retaliate against Palestine because of its military and technological capabilities. Palestine on the other hand has required long time and planning to carry out any attack against Israel because of its limited technological and military resources[4]. So, even if Palestine strongly wants to retaliate against Israel, it cannot because of its lack of military capability.

In a game-theoretic setup, revenge has been modelled as an increasing function of the opponent's conflict investment of the previous period (Amegashie and Runkel, 2012;

[2]Bolle et al. (2016) experimentally showed the real evidence of revenge. They showed that two-third of the games end up with the players engaging in violent behaviour until they had the lowest possible winning probabilities.

[3]The 2019-2020 Persian Gulf crisis is an example of this conflict escalation out of revenge. See Section 4.7.2.

[4]See Section 4.7.3.

Liang et al. 2020). But in reality, the motive of a combatant to go into a second period conflict out of revenge depends on its military capability to do so. This chapter fills in this gap and analyses the strategic behaviour of the combatants and its impact on the intensity of the conflict.

As per Amegashie and Runkel's (2012) formulation, the revenge function in the current period is an increasing function of the opponent's conflict investment of the previous period. This chapter models a revengeful conflict in game-theoretic setup where a combatant's desire to go into a second period conflict out of revenge depends on its capability to do so, it is called a 'revenge-capability' function. The 'revenge-capability' function captures the interaction between the desire to revenge and the capability to do so. Specifically, it shows that a combatant's amount of revenge this period not only depends on its opponent's conflict investments in the previous period, but also on its own resource capability that is available with it, given destruction suffered in the previous period.

4.1.1 Related Literature:

This model is a reformulation of Amegashie and Runkel's (2012) game theoretic two-period model of conflict with revenge. Compared to their model this model not only incorporates the desire factor of revenge but also takes into account the capabilities of the combatants to exact revenge in the second period.

The idea of military capability is not new in the context of defender-attacker conflict scenario. The balance of military capabilities between the attacker and the defender is given the most attention to analyse deterrent situations. The attacker must evaluate whether it has sufficient military capability or power to win a conflict with the defender and at what cost (Huth and Russett 1984, 1988).

Dixit (1987) in his paper showed that the favourite or the player in a favorable position in the contest will commit effort at a higher level than that in a Nash equilibrium and the player not in a favorable position (the underdog) will commit at a lower level. We get Dixit's (1987), result even in this two-period game of conflict. A combatant who can partially incapacitate its opponent in the first period conflict is in a favorable position in the conflict and thus will increase its first period effort and can prevent its opponent from going into second period conflict.

In their empirical analysis of the Second Intifada, Jaegar and Paserman (2008) found that Israel significantly reacts to Palestinian violence against them whereas Palestine's actions are not significantly related to Israeli violence against them. Israel is a country with organizational and technical capabilities and is thus able to respond in a conflict when it wishes. However, Palestine is a country with limited means, thus responding against Israel requires long planning and complicated logistics. In short, it shows that Israel is in a favourable position in the conflict compared to Palestine. In this chapter, it has been shown that the stronger combatant in the conflict will increase its first period conflict effort and prevent its opponent from retaliating out of revenge, which also explains Palestine's lack of response to Israel's violence against them.

This model shows that there is a significant incapacitation effect when both the combatants are strong enough to partially incapacitate each other in the first period. Hence, after the first period conflict both the combatants will be unwilling to go into a second period conflict out of revenge (due to incapacitation), thus leading to de-escalation of the conflict. The results of this model resonates with the 'The Balance of Power' model. 'The Balance of Power' model is based on the trinity of beliefs: equality of power is conducive to peace; an imbalance of power leads to war; the stronger party is the probable aggressor (Organski and Kugler, 1980).

The rest of the chapter is organized as follows; section 4.2 presents the model, section 4.3 discusses the revenge-capability function, section 4.4 presents the game and discusses the effects of revenge on the strategies of the combatants and thus, on the level of conflict, section 4.5 presents the subgame perfect Nash equilibrium (SPNE) of the game when capabilities of the combatants are equal (complete symmetric case), section 4.6 discusses about the equilibrium conflict investments of the combatants and the level of the conflict when the capabilities of the combatants are different (asymmetric case), section 4.7 discusses some real life conflicts where we can see these effects of revenge on the strategies of the combatants and on the level of conflict and section 4.8 concludes.

4.2 The Model

There are two combatants, A, B engaged in a conflict over a resource. Let the value of the resource be V_i where $i = A, B$. We assume that $V_A = V_B = V$[5]. The combatant who wins the conflict gets the resource. The combatants can engage in second period conflict out of revenge for the damages/injuries it suffered in the first period.

[5]This assumption is important because in order to analyse all the effects of the presence of revenge on the strategic behaviour of the combatants we need to ensure that the valuation difference of the resource won't put any additional pressure on the effects of revenge. Amegashie and Runkel (2012) showed in an asymmetric case when the valuation of the resource is different for the combatants then the self-deterrence effect is dominant over the value-effect of revenge. Caruso (2006, 2007) showed that asymmetry in the combatants' evaluation of the stake can act as a powerful force influencing the strategic behaviour of the combatants.

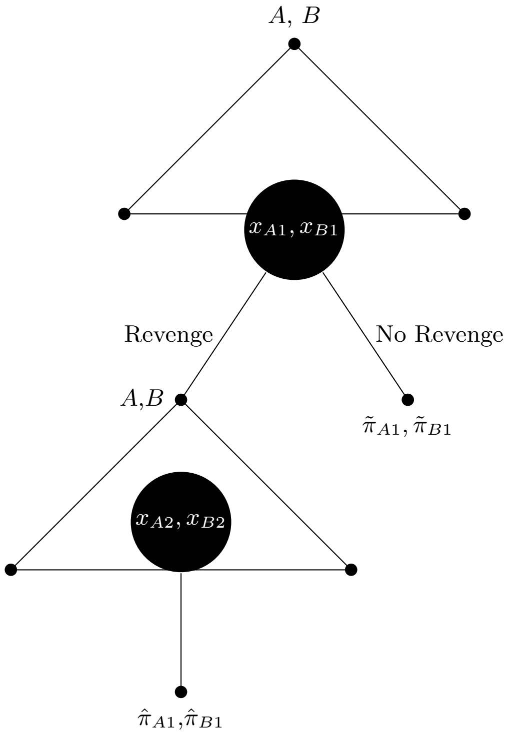

FIGURE 4.1: Two period conflict between two combatants in the presence of revenge motivations.

Figure 4.1 shows the game tree, where the first period conflict takes place for the resource and the second period conflict out of revenge.

The probability with which player i wins the conflict in period t is given by the standard ratio-form Tullock (1980) contest success function:

$$P_{it} = \frac{x_{it}}{x_{it} + x_{jt}}, i, j = A, B, i \neq j, t = 1, 2 \tag{4.1}$$

where x_{it} is conflict investment of combatant i at time t. The cost function is assumed to be a linear function of conflict investment and the marginal cost is also normalised to 1, for simplicity. Thus, the cost function of the combatant i at time t is $C(it) = x_{it}$ where, x_{it} is the conflict investment of combatant i at time t.

As a benchmark, we first analyse the value of the conflict without revenge.

4.2.1 Subgame with no second period: Conflict without Revenge

It is a complete information single period game[6]. In the absence of revenge, there is no second period conflict, thus no second period conflict investments. Letting all variables in this case be denoted by tilde, we have $\tilde{x}_{A2} = \tilde{x}_{B2} = 0$. Let $\tilde{X}_2$ be the aggregate revenge investments in period 2. Hence, $\tilde{X}_2 = \tilde{x}_{A2} + \tilde{x}_{B2} = 0$. In period 1, the payoff function of player $i, j = A, B, i \neq j$, is given by the following:

$$\pi_{i1} = P_{i1}V - x_{i1}. \tag{4.2}$$

Maximising w.r.t. x_{i1}, and assuming an interior optimum, we arrive at the equilibrium levels of conflict investments, $\tilde{x}_{A1}, \tilde{x}_{B1}$. Hence, we can find the aggregate conflict investment in period 1, $\tilde{X}_1 = \tilde{x}_{A1} + \tilde{x}_{B1}$. There is no second period conflict, so the aggregate level of conflict is the aggregate level of conflict in period 1. The best response function of the combatants is, $x_{i1} = \sqrt{Vx_{j1}} - x_{j1}$ where $i, j = A, B$ and $i \neq j$. The equilibrium conflict investments are unique and symmetric, $x_{A1} = x_{B1} = \frac{V}{4}$, (Chowdhury and Sheremeta, 2011). The aggregate level of conflict is $\tilde{X} = \frac{V}{2}$.

4.3 The Revenge-Capability Function

Now, A and B are engaged in revengeful conflict with each other. Initially there has been a huge attack by party B on party A. Now A's revenge in the next period, according to the Amegashie and Runkel (2012) formulation, would be huge. However, because of this huge attack, it is possible that A has suffered irreparable damage in the first period and hence is simply unable to launch a huge revenge-motivated attack on B in the next period. In other words, it is possible that A has been substantially incapacitated so as not to be able to launch a massive counter attack on B as desired. The revenge-capability function of a combatant shows that the desire to go into a second period conflict out of revenge depends on the state of its capability to do so, given the destruction suffered in the first period.

Let R_i be the revenge-capability function of combatant i. R_i is basically prize value combatant i gets after the second period conflict (out of revenge). Now, this prize value depends on the excess resources it has (after the first period conflict) to go into a second period conflict out of revenge. If the combatant has suffered substantial destruction in the first period conflict then the combatant would prefer to not go into a second period

[6] This is essentially from the section 'Equilibrium without Revenge' in the Amegashie and Runkel (2012) paper (pg. 317).

conflict out of revenge (even if it wants to) because of its limited military capability[7]. The '*revenge-capability function*' captures the interaction between desire and capability of a combatant to go into a second period conflict out of revenge. The two factors, desire and capability to revenge are explained below:

1) The desire for revenge (satisfaction from exacting revenge) of the combatants will be quantified by the amount of first period destruction suffered by the combatant in the hands of its enemy. The first period conflict investment of the opponent of the combatant is the proxy variable for the destruction suffered by the combatant in the first period, Amegashie and Runkel (2012). Thus, x_{j1} is the 'quantity of revenge' that i wants to exact from j, $i \neq j$. Eg: Combatant A suffers a destruction of x_{B1} in the first period. x_{B1} (conflict investment of B in the first period conflict) captures the fatalities of civilians and military personnel and structural/military damages or destruction. Combatant A would want to exact revenge for every amount of the destruction suffered in the hands of its opponent[8].

2) The capability of combatant i to retaliate can be showed with the function $(\alpha_i - \gamma_i x_{j1})$, where $\alpha_i > 0$ and $0 < \gamma_i < 1$.

The capability function is explained in the following way:

2.1) α_i captures the 'strength' of combatant i. The word strength captures the structural, economic and most importantly military resources of combatant i. The higher the value of α, stronger the combatant.

2.2) γ_i is the fraction of resources lost in the conflict. The first period conflict investment of the opponent of the combatant is the destruction suffered by the combatant in the first period. $\gamma_i x_{j1}$ captures the amount of structural/military damages combatant i suffered in the first period, where $i \neq j$. Lower the value of γ, less the structural/military damages suffered.

Thus, $(\alpha_i - \gamma_i x_{j1})$ is the excess of (military and structural) resources available to combatant i to retaliate in the second period[9].

[7]Rabin's theory says that we can expect to see a reciprocal behaviour such that harmful or aggressive actions are met with harmful actions given that the stakes are not so high (Jennings, 2016). This revenge-capability function tries to capture this idea, that is if the combatant has military capability to go into a conflict out of of revenge then only it will engage into a conflict out of revenge.

[8]Löwenheim and Heimann (2008) considers revenge to be a form of negative reciprocity and it is responsive in nature. They argue that a state will become more revengeful when; they strongly sense the harm they received as morally outrageous; their intense feelings of humiliation and the less institutionalized norms that govern negative reciprocity in international politics.

[9]This resembles with the classic idea of guns and butter. Skaperdas (1992) showed that *ceteris paribus* the more powerful agent possesses less valuable productive resources and invests more in arms. Caruso and Echevarria-Coco (2022) theoretically and empirically showed that the international prices of manufactures are negatively associated with arms imports and military expenditure in the context of sub-Saharan African countries for the period 1980-2017.

The revenge-capability function is the interaction of these two factors, 'desire to exact revenge' and 'capability to exact revenge'[10]. That is the desire to revenge depends on the state of the capability to revenge[11].

The revenge-capability function is:

$$R_i = (x_{j1}) * (\alpha_i - \gamma_i x_{j1}) \geq 0, i \neq j \tag{4.3}$$

In short, there is no use of one without the other, that is if the combatant has desire to exact revenge but no capability then there won't be any second period conflict. Similarly, if it has no desire but has capability then also there won't be any second period conflict[12].

Now, differentiating the revenge-capability function w.r.t x_{j1}, we get;

$$\frac{dR_i}{dx_{j1}} = \alpha_i - 2\gamma_i x_{j1} \lesseqgtr 0, \quad \text{when} \quad x_{j1} \gtreqless \frac{\alpha_i}{2\gamma_i}. \tag{4.4}$$

[10]Garfinkel and Skaperdas (2007) in their model showed that if there is a conflict and fighting occurred then a fraction of the resource (for which the conflict takes place) will be left for the winner. If $\bar{S}$ is the resource then fighting leads to destruction of δ fraction of $\bar{S}$ and the winner gets $(1-\delta)\bar{S}$ of the resource. If δ is very high then the combatants would be better-off with settlement (Anderton and Carter, 2009). $(1-\delta)\bar{S}$ is the fraction of resource left over after the conflict for the winner. Similarly, in this setup capability is basically the excess amount of resources left after the first-period conflict to go into a second-period conflict out of revenge. Here, capability of a combatant is endogenous and depends on the first period conflict investment of the opponent.

[11]If we consider the case, a sequential game where the combatants engage into a second period conflict for the same resource (V) like in the first period, then the combatants would consider the discounted value of the resource i.e., their second period prize value will be $r.V$ where, r is the discount rate (r is same for the combatants). In short, the value of the resource may fall (or rise) in the second period for the combatants. This change in the valuation of the resource (of the combatants) is captured by the discount rate. If the rate of discount r is less than one, then the value of the resource for the combatants falls after every conflicts. In this model, the combatants are engaging into a second period conflict out of revenge. Revenge is a sense of satisfaction, a sense of honor, it does not hold any material value like the value of the resource for which the conflict takes place (V). So, when a combatant engages into a (second) conflict for something that is not materialistically valued, the combatant will take into account its excess military capabilities after the first period conflict. The capability function captures the fact that how much it is worth for a combatant to go into a second period conflict out of revenge after the destruction suffered in the first period. In short, if a combatant has suffered massive damages and destruction then the outcome of engaging into a second conflict out of revenge will be very small. The capability function tries to capture this change in the valuation of revenge (of the combatants). If the combatant is very strong (high value of α given the damages suffered in the first period) then the value of revenge for that combatant (in the second period) would be high i.e., the combatant would highly prefer to take revenge.

[12]Amegashie and Runkel (2012) discussed a formulation of revenge where the desire for revenge is elastic (or inelastic) to the destruction suffered in the previous period. As per this model, the revenge function becomes; $R_i = x_{j1}^{\phi} * (\alpha_i - \gamma_i x_{j1})$. Since this formulation of revenge does not change the results, it has been discussed in Appendix B.0.1 of the dissertation.

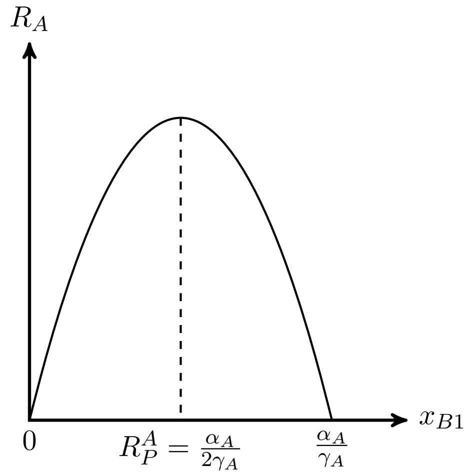

FIGURE 4.2: Revenge-Capability function of A as a function of x_{B1}.

So A's revenge-capability function will be;

$$R_A = x_{B1}(\alpha_A - \gamma_A x_{B1}) > 0 \text{ when } 0 < x_{B1} < \frac{\alpha_A}{\gamma_A}. \tag{4.5}$$

$$R_A \leq 0 \text{ when } x_{B1} \geq \frac{\alpha_A}{\gamma_A} \implies \text{Full incapacitation of } A. \tag{4.6}$$

$$R_A \leq 0 \implies R_A = 0. \tag{4.7}$$

Similarly for combatant B.

Equation (4.5) shows the range when there will be a positive desire and capability for A to exact revenge.

Equation (4.6) shows that when the combatant has been fully incapacitated, then there will be a positive desire to exact revenge but zero capability to retaliate, which makes R_A zero. Thus, when $x_{B1} \geq \frac{\alpha_A}{\gamma_A}$ combatant B has totally incapacitated its opponent A in the first period conflict and A won't retaliate in the second period out of revenge[13].

Figure 4.2 is the revenge-capability function of combatant A. When $x_{B1} \geq \frac{\alpha_A}{2\gamma_A}$, combatant A's value of R_A is falling thus the incentive to retaliate (out of revenge) starts falling when the destruction it suffered is more than R_P^A. Let this point of destruction suffered by combatant i in the first period, beyond which combatant i's revenge starts falling be denoted by R_P^i, 'P' for peak point of the revenge-capability function[14]. In

[13] This is the point of complete incapacitation after which the conflict ends. Japan was completely incapacitated after the atomic bombing in Hiroshima and Nagasaki (See Section 4.7.1).

[14] Here, the revenge-capability function looks like an inverted-U curve, which basically shows that revenge for a combatant will rise till a certain point but after that it will start falling because of incapacitation. The inverted-U curve is a prevalent curve in economics, for example the classic Kuznets curve. The Kuznets curve expresses a hypothesis that as an economy develops, the economic inequality first rises and then falls (Kuznets, 1955).

short, when x_{j1} is more than $R^i_P = \frac{\alpha_i}{2\gamma_i}$, it implies that j has incapacitated its opponent, i to its maximum revenge point. Thus, i's revenge-capability function is falling with higher x_{j1}.

4.4 Subgame Equilibrium - Conflict with Revenge

It is a complete information simultaneous game, both the combatants knows the conflict investments of its opponents. We proceed with backward induction in order to calculate the subgame-perfect equilibrium conflict investment. We first calculate the second period equilibrium conflict investments. The second period conflict out of revenge only takes place when both the combatants have positive desire and capability to exact revenge[15].

Let $x_{i2}, i = A, B$ denote the efforts expended by the combatants in the 2nd period conflict out of revenge. The 2nd period payoff functions are:

$$\pi_{A2} = P_{A2}R_A - x_{A2}. \tag{4.8}$$

$$\pi_{B2} = P_{B2}R_B - x_{B2}. \tag{4.9}$$

Maximising the payoff functions[16], we get 2nd period equilibrium conflict investments;

$$\hat{x}_{i2} = \frac{R_i^2 R_j}{(R_A + R_B)^2}, \quad \text{where } \ i \neq j \ \text{ and } \ i, j = A, B. \tag{4.10}$$

$$\hat{X}_2 = \hat{x}_{A2} + \hat{x}_{B2} = \frac{R_A R_B}{(R_A + R_B)}. \tag{4.11}$$

The second period conflict investments are a function of the first period conflict investments of both the combatants. Equation (4.10) is **the value-effect or vengeance effect of revenge**.

[15] In this setup, the constraint is $x_{j1} < \frac{\alpha_i}{\gamma_i}$ where $i \neq j$. The constraint is independent of the choice variable of combatant i in the second period. If we break the payoff function of combatant i in terms of constrained optimization, then it looks like,

$$\max \pi_{i2} = P_{i2}x_{j1} - x_{i2}$$
$$s.t \ \ P_{i2}x_{j1}(\alpha_i - \gamma_i x_{j1}) \geq P_{i2}x_{j1}$$

Here, the first equation is the payoff function of combatant i as per Amegashie and Runkel's (2012) formulation. The constraint solves into $\alpha_i > 1 + \gamma_i x_{j1}$ which implies $\alpha_i > \gamma_i x_{j1}$.

[16] If we break the payoff functions, then it looks like, $\pi_{i2} = P_{i2}(\alpha_i x_{j1}) - x_{i2} - P_{i2}(\gamma i x_{j1}^2)$, where $i \neq j$. It is evident that the opponent's first period conflict investment increases the cost of conflict for the combatant in the second period. This somewhat coincides with the idea of 'sabotage'. Sabotage is the intentional and costly act of damaging the opponent's chance of winning the contest. Chowdhury and Gurtler (2015) give a detailed survey of sabotage in contests.

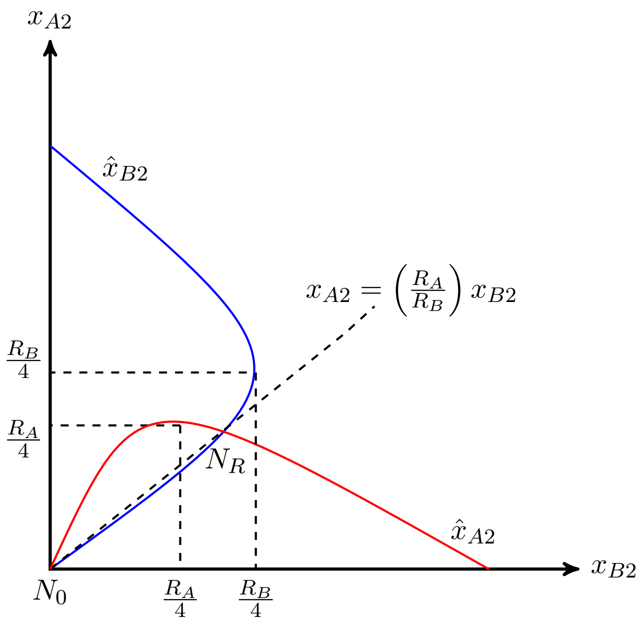

FIGURE 4.3: Best response functions of the combatants in period-2 and Nash Equilibrium.

The period-2 game has two equilibria. The period-2 best response functions of the combatants can be written in the following way[17].

$$\hat{x}_{A2} = \sqrt{R_A x_{B2}} - x_{B2}. \tag{4.12}$$

$$\hat{x}_{B2} = \sqrt{R_B x_{A2}} - x_{A2}. \tag{4.13}$$

Figure 4.3 shows the best-response functions of the combatants in period-2[18], where $\hat{x}_{A2}$ is the best response function of combatant A (equation (4.12)) and $\hat{x}_{B2}$ is the best response function of combatant B (equation (4.13)). The curves intersect twice at N_0 and at N_R. N_0 is the no-revenge equilibrium and N_R is the revenge equilibrium. N_R is a stable equilibrium with respect to Cournot stability concept[19], since any deviation from the origin would trigger a chain reaction of the two combatants that will only stop at N_R[20]. However if the two combatants managed to cooperate by some unspecified

[17] It is derived from the first order conditions of the maximization problems (4.8) and (4.9), as shown below;
$\frac{d\pi_{i2}}{dx_{i2}} = \frac{R_i x_{x_{j2}}}{(x_{A2}+x_{B2})^2} - 1 = 0, \;\; i \neq j, \;\; i, j = A, B.$

[18] Figure 4.3 is for interpretation only, where $R_A < R_B$. It can also be the case where $R_A > R_B$.

[19] It can be explained in the following way; let the combatants be at N_0 in figure 4.3, suppose if there is any increase in x_{B2} from 0, then from $A's$ reaction function we get a positive x_{A2}. Then corresponding to that value of x_{A2} there is a higher value of x_{B2} from $B's$ reaction function. This adjustment process will continue till it reaches N_R. Similarly, if x_{A2} increases from 0.

[20] This might be viewed as the new *Nemesis*, the goddess that goaded the ancient Greeks to seek revenge.

mechanism (Schelling's (1960) 'focal point' hypothesis[21]) and chose simultaneously then the no-revenge equilibrium would prevail. It can be viewed as a coordination failure problem[22].

Putting the value of the 2nd period equilibrium conflict levels in the payoff function;

$$\hat{\pi}_{i2} = \frac{R_i^3}{(R_A + R_B)^2}, \quad \text{where} \quad i = A, B. \tag{4.14}$$

For the analysis to follow, we need to know the impact of combatant i's first period effort on combatant i's second period equilibrium payoff.

$$\frac{d\hat{\pi}_{A2}}{dx_{A1}} = -\frac{2R_A^3 R_B'}{(R_A + R_B)^3} \lesseqqgtr 0, \quad \text{when} \quad R_B'(x_{A1}) \gtreqqless 0 \implies x_{A1} \lesseqqgtr \frac{\alpha_B}{2\gamma_B}. \tag{4.15}$$

$$\frac{d\hat{\pi}_{B2}}{dx_{B1}} = -\frac{2R_B^3 R_A'}{(R_A + R_B)^3} \lesseqqgtr 0, \quad \text{when} \quad R_A'(x_{B1}) \gtreqqless 0 \implies x_{B1} \lesseqqgtr \frac{\alpha_A}{2\gamma_A}. \tag{4.16}$$

From equation (4.15) and (4.16) we see that the impact of combatant i's first period effort on its second period equilibrium payoff depends on the differentiated revenge-capability function of its opponent. The second period payoff of combatant A is increasing w.r.t to its first period conflict investment when the first period conflict investment of the combatant is more than the R_P^B $(= \frac{\alpha_B}{2\gamma_B})$ point of its opponent. It is decreasing w.r.t to A's first period conflict investment when the first period conflict investment of the combatant is less than the R_P^B $(= \frac{\alpha_B}{2\gamma_B})$ point of its opponent. The payoff function in period 1 is the present value (without discounting) of the payoffs from both the periods, as given below:

$$\pi_{i1} = \frac{x_{i1}}{x_{A1} + x_{B1}} V - x_{i1} + \hat{\pi}_{i2}, \quad \text{where} \quad i = A, B. \tag{4.17}$$

Maximizing the payoff functions w.r.t x_{i1}, the FOCs are;

$$\frac{d\pi_{i1}}{dx_{i1}} = \frac{x_{j1}}{(x_{A1} + x_{B1})^2} V - 1 + \frac{d\hat{\pi}_{i2}}{dx_{i1}} = 0, \quad \text{where} \quad i \neq j \quad \text{and} \quad i = A, B. \tag{4.18}$$

[21]The 'focal point' hypothesis of Shelling (1960) argues that a successful solution of implicit coordination problems depends on whether the expectations of the players can converge to an outcome, the focal point. Chowdhury et al. (2020) experimentally examined the influence of 'focality' on individual behaviour in a class of multi battle contests where the Nash equilibrium is unique and in pure strategies.

[22]If we delve a bit deeper, then this second-period coordination problem entails a credible commitment problem since the players cannot tie their hands in the first period that they won't revenge in second-period. Thus this reminds us of the commitment-failure problem (Azam, 1995; Fearon, 1995 and Blattman, 2023). Kreps et al. (1982) showed how informational asymmetries can have a reputation effect which can generate cooperative behaviour in the classic finitely repeated prisoners' dilemma game.

Observation 4.1. When $\frac{d\hat{\pi}_{i2}}{dx_{i1}} < 0$, that is when $x_{i1} < \frac{\alpha_j}{2\gamma_j}$, for $i, j = A, B, i \neq j$, we observe the **self-deterrence effect**. The self-deterrence effect reduces the first period conflict investment of the combatant (compared to the no-revenge case)[23].

Observation 4.2. When $\frac{d\hat{\pi}_{i2}}{dx_{i1}} > 0$, that is when $x_{i1} > \frac{\alpha_j}{2\gamma_j}$, for $i, j = A, B, i \neq j$, there won't be any self-deterrence of combatant i. When $x_{i1} > \frac{\alpha_j}{2\gamma_j}$ (combatant i can reach its opponent's revenge peak point), combatant i will have a favourable position in the conflict (i is the stronger combatant in the conflict) and thus will increase its first period conflict investment (compared to the no-revenge case)[24] so that it can dis-incentivize its opponent j to go into second period conflict out of revenge through incapacitation effect[25] (See Section 4.4.0.1).

Now, let us solve for the equilibrium conflict levels;

$$\frac{d\pi_{i1}}{dx_{i1}} = \frac{x_{j1}V}{(x_{A1} + x_{B1})^2} - 1 - \frac{2R_i^3(\alpha_j - 2\gamma_j x_{i1})}{(R_A + R_B)^3} = 0, \quad \text{where } i \neq j \text{ and } i = A, B. \quad (4.19)$$

The FOCs can be written in the following way;

$$x_{i1} = \frac{x_{B1}x_{A1}V}{(x_{A1} + x_{B1})^2} - \frac{2R_i^3 x_{i1}(\alpha_j - 2\gamma_j x_{i1})}{(R_A + R_B)^3}, \quad \text{where } i \neq j \text{ and } i = A, B. \quad (4.20)$$

Total conflict in this setup, $X_R = x_{A1} + x_{B1} + \frac{R_A R_B}{(R_A + R_B)}$. Now, in order to show whether the paradox of revenge holds or not, we must compare the conflict level with revenge to conflict level without revenge, $X_R - \tilde{X}$. Here, the paradox of revenge is from Amegashie and Runkel's (2012) concept, when self-deterrence effect is more than value-effect such that the conflict level with revenge is less than conflict level without revenge. From section 4.2.1 we know that the aggregate level of conflict without revenge is $\tilde{X} = \frac{V}{2}$.

Proposition 4.1. *The paradox of revenge can be observed ($X_R < \tilde{X}$), when $x_{A1} < \frac{\alpha_B}{3\gamma_B}$, $x_{B1} < \frac{\alpha_A}{3\gamma_B}$, and $R_A > R_B$ such that $R_B^2 < R_A x_{A1}(\alpha_B - 3\gamma_B x_{A1})$. Similarly, when $R_B > R_A$.*

Proof. Proof in Appendix B.0.2. ■

[23] From (4.18) we can see that compared to the no-revenge payoff function of the combatant there is an additional negative term, $\frac{d\hat{\pi}_{i2}}{dx_{i1}}$. Which implies that the maximization of the payoff function of the combatant happens at a lower value of x_{i1}.

[24] From (4.18) we can see that compared to the no-revenge payoff function of the combatant there is an additional positive term, $\frac{d\hat{\pi}_{i2}}{dx_{i1}}$. Which implies that the maximisation of the payoff function of the combatant happens at a higher value of x_{i1}.

[25] 1) Dixit's (1987) result, the favourite commits effort at a higher level than that in the Nash equilibrium and the underdog at a lower level. 2) The 'Balance of Power' model states that the stronger party is the most probable aggressor in a conflict (Organski and Kugler, 1980).

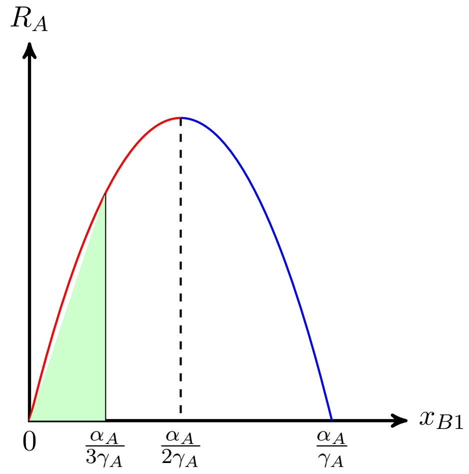

FIGURE 4.4: A's level of revenge depending on B's first period conflict investment.

Observation 4.3. The paradox can be observed only when the incapacitation effect is low enough (for both of the combatants) for the self-deterrence effect to come into play.

Observation 4.4. When both the combatants cannot incapacitate each other to a certain level in the first period conflict and also when the revenge motivation of one of the combatants is relatively very high, then that combatant has a high incentive to strike back or retaliate. This leads to a decrease in effort of its opponent. Thus, the self-deterrence effect of the less revengeful combatant dominates the value-effect of the more-revengeful combatant.

In short, we can also say that when both the combatants are not able to incapacitate each other to a large extent, we may notice higher self-deterrence effect than the value effect.

Figure 4.4 shows A's level of revenge depending on its opponent, B's first period conflict investment. When $x_{B1} \in (0, \frac{\alpha_A}{3\gamma_A})$ the paradox of revenge may hold. From Proposition 1, we know that it must hold for both the combatants, that is the first period conflict investment of both the combatants must be less than $\frac{\alpha_j}{3\gamma_j}$ where, j stands for the opponent of the combatant. The revenge-capability function (of combatant A) is rising with x_{B1} when $x_{B1} < \frac{\alpha_A}{2\gamma_A}$ and we observe self-deterrence effect of combatant B. The revenge-capability function (of combatant A) is falling with x_{B1} when $x_{B1} > \frac{\alpha_A}{2\gamma_A}$, thus, B has partially incapacitated A, thus no self-deterrence effect of B.

Let us check how second period equilibrium conflict investments are motivated by first period conflict investments.

4.4.0.1 Impact of the opponent's first period conflict effort on the combatant's second period equilibrium conflict effort

$$\frac{d\hat{x}_{i2}}{dx_{j1}} = \frac{2R_j^2 R_i R_i^{'}}{(R_A + R_B)^3} \gtrless 0, \text{ when } R_i^{'} \gtrless 0 \implies x_{j1} \lessgtr \frac{\alpha_i}{2\gamma_i}, \text{ where } i \neq j. \tag{4.21}$$

Let us interpret equation (4.21) in terms of A and B, where $i = B$ and $j = A$. then we get the following equation;

$$\frac{d\hat{x}_{B2}}{dx_{A1}} = \frac{2R_A^2 R_B R_B^{'}}{(R_A + R_B)^3} \gtrless 0, \text{ when } x_{A1} \lessgtr \frac{\alpha_B}{2\gamma_B}. \tag{4.22}$$

When x_{A1} is more than $\frac{\alpha_B}{2\gamma_B}$ the (equilibrium) conflict investment of B in the second period, $\hat{x}_{B2}$ falls. This is called the **incapacitation effect**. The incapacitation effect reduces the second period conflict investment of the combatants.

Observation 4.5. When combatant A's first period conflict effort is more than R_P^B $(= \frac{\alpha_B}{2\gamma_B}$, B's revenge-capability function), combatant B will reduce its second period conflict investment because combatant B has been partially incapacitated in the conflict and going into a second period conflict out of revenge will cause further destruction and thus combatant B will be less willing to go into a second period conflict out of revenge.

4.4.0.2 Impact of the combatant's first period conflict effort on the combatant's second period equilibrium conflict effort.

$$\frac{d\hat{x}_{B2}}{dx_{B1}} = \frac{R_B^2 R_A^{'}(R_B - R_A)}{(R_A + R_B)^3}. \tag{4.23}$$

Equation (4.23) is very interesting, it captures change in combatant B's second period (equilibrium) conflict investment due to change in its first period conflict investment other things being constant.

Observation 4.6. Now, if $R_A' < 0$ (when $x_{B1} > \frac{\alpha_A}{2\gamma A}$), B has almost incapacitated A, then there will be reduction in combatant B's second period conflict investment only when $R_B > R_A$ and it will increase its effort otherwise ($R_A > R_B$).

It can be explained in the following way: When B's first period conflict investment is above R_P^A it means that A's revenge motivation is falling with more destruction suffered in the first period but the value of the revenge can still be high. Given B's capability if it is able to incapacitate its opponent (A) to such a point that its revenge is falling but at the same time it is relatively high then combatant B will increase its conflict investment in the second period, since A may still want to go in a second period conflict out of revenge[26].

Observation 4.7. When the revenge motivation of A will be less than B, combatant B will reduce its second period conflict investment because it will know that A would prefer not to go into a second period conflict out of revenge as A is comparatively less-revengeful than B thus combatant B will reduce its conflict investment.

4.5 SPNE - Symmetric Case

In this section, we calculate the equilibrium conflict investments of the combatants and the subgame perfect Nash equilibrium (SPNE) when the capabilities of the combatants are equal, that is for the symmetric case.

Proposition 4.2. *When $\alpha_A = \alpha_B = \alpha$ and $\gamma_A = \gamma_B = \gamma$:*
i) The conflict investments of both the combatants are equal,
$\hat{x}_{A1} = \hat{x}_{B1} = \hat{x}_1 = \frac{V}{4} - \frac{\hat{x}_1(\alpha - 2\gamma\hat{x}_1)}{4}$.
ii) There will be an increase in the total level of conflict (X_R) depending on γ, $X_R = \frac{V}{2} + \frac{\hat{x}_1^2\gamma}{2}$.

Proof. Proof in Appendix B.0.3. ■

Now, the question arises does both the countries first period conflict investment is more or less than $\frac{\alpha}{2\gamma}$. The answer lies whether the value of the resource V for which the conflict takes place in the first place is more or less than $\frac{2\alpha}{\gamma}$. The result can be shown in the following way;

Let $x_{A1} = x_{B1} = \frac{\alpha}{2\gamma}$, then the first order conditions in equation (4.19) becomes;

$$\frac{d\pi_{A1}}{dx_{A1}} = \frac{d\pi_{B1}}{dx_{B1}} = \frac{\gamma V}{2\alpha} - 1 \tag{4.24}$$

Now, if $V > \frac{2\alpha}{\gamma}$ then equation (4.24) is positive which implies that when $x = \frac{\alpha}{2\gamma}$ the FOC is positive then the value of the first period conflict investment must increase in order

[26]It is evident from the formulation of the revenge function, higher first period conflict investment of B leads to higher desire factor of revenge of A. So the even though the slope of the revenge function (of A) is falling, the value of revenge can still be high for A to go into a second period conflict out of revenge. See Section 4.5 where the SPNE in the symmetric case shows the same.

to obtain optimum for both the countries. And if $V < \frac{2\alpha}{\gamma}$ then the FOC is negative and thus first period conflict investment must decrease in order to obtain optimum. The paradox of revenge won't be observed in this case since the total level of conflict rises for any value of the resource (V).

Proposition 4.3. *When $\hat{x}_1 < \frac{2\alpha}{3\gamma}$, the combatants are better off in the presence of revenge. The SPNE in this case will be to take revenge. When $\hat{x}_1 > \frac{2\alpha}{3\gamma}$, the combatants are worse-off in the presence of revenge. The SPNE in this case will be to not take revenge.*

Proof. Proof in Appendix B.0.4. ■

The combatants are worse-off in the presence of revenge, when $\hat{x}_1 > \frac{2\alpha}{3\gamma}$. The SPNE in this case will be to not take revenge. Figure 4.5 shows the SPNE, when the combatants will not go into a second period conflict out of revenge in a complete symmetric case. When $\hat{x}_1 < \frac{2\alpha}{3\gamma}$, the combatants are better off in the presence of revenge. The SPNE in this case will be to take revenge. Figure 4.6 shows the SPNE when the combatants will go into second period conflict out of revenge in a complete symmetric case.

It can also be a case such that $\frac{\alpha}{2\gamma} < \hat{x}_1 < \frac{2\alpha}{3\gamma}$, like mentioned in section 4.4.0.2, the revenge function beyond this point is falling with the destruction suffered in the first period but the value of revenge can be a high, thus when $\hat{x}_1$ lies within this range there will be a second period conflict out of revenge but the intensity of the second period conflict will be low because of the incapacitation effect.

4.6 Asymmetric Case

In this section we discuss about the equilibrium conflict investments of the combatants and the level of the conflict when the capabilities of the combatants are different.

In this framework the difference in the first period conflict investments of the combatants can be explained by the difference in their capabilities.

If we simplify (4.20) then we can write the conflict investments of the combatants in terms of the other which gives the following equation[27],

$$\hat{x}_{A1}(1 + K_1) = \hat{x}_{B1}(1 + K_2), \tag{4.25}$$

[27] Simplification in Appendix B.0.5.

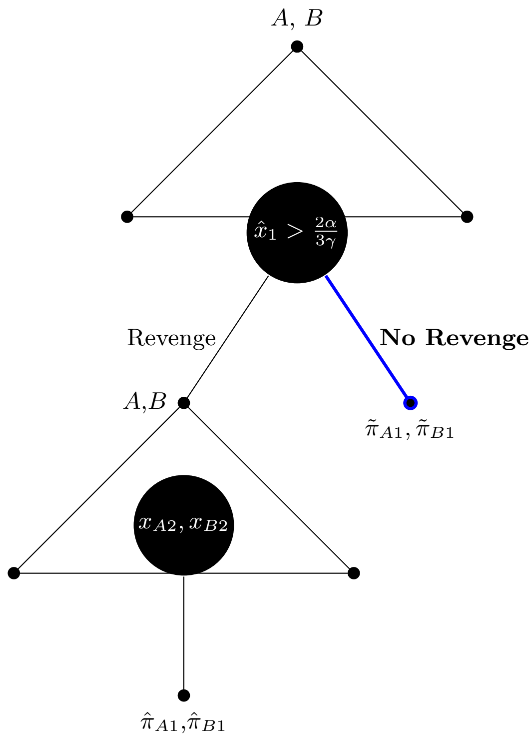

Figure 4.5: SPNE when the combatants are better off without revenge in a complete symmetric case.

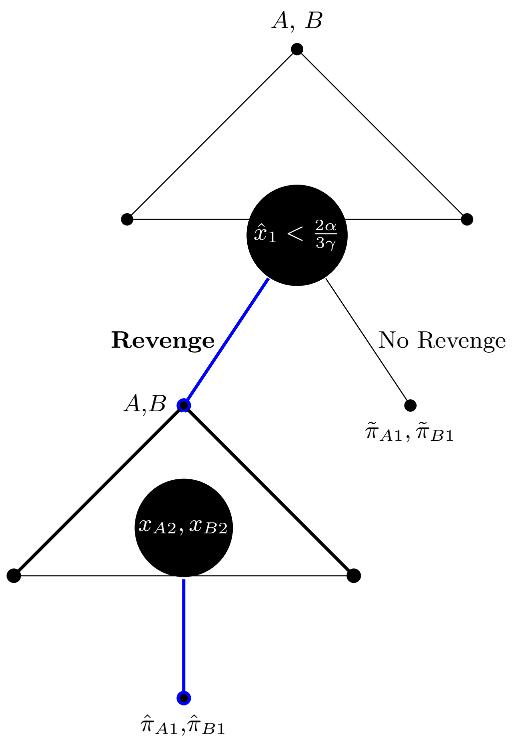

Figure 4.6: SPNE when the combatants are better off with revenge in a complete symmetric case.

where, $K_1 = \frac{2R_A^3 R_B^{'}}{(R_A+R_B)^3}$ and $K_2 = \frac{2R_B^3 R_A^{'}}{(R_A+R_B)^3}$.

Using equation (4.25) Proposition 4.4 summarises how the first period conflict investments of the combatants vary with respect to each others capabilities.

Proposition 4.4. *When $\frac{\alpha_B}{\gamma_B} > \frac{\alpha_A}{\gamma_A}$, the first period conflict investment of B is more than that of A, $\hat{x}_{B1} > \hat{x}_{A1}$. Similarly, when $\frac{\alpha_B}{\gamma_B} < \frac{\alpha_A}{\gamma_A}$, $\hat{x}_{B1} < \hat{x}_{A1}$.*

From (4.25) it is evident that $\hat{x}_{B1} > \hat{x}_{A1}$ when $K_1 > K_2$. And $K_1 > K_2$ when $R_A^3 R_B^{'} > R_B^3 R_A^{'}$.

Observation 4.8. $R_A^3 R_B^{'} > R_B^3 R_A^{'}$ strictly holds when $R_B^{'} > 0$ and $R_A^{'} < 0$ i.e., when $\hat{x}_{A1} < \frac{\alpha_B}{2\gamma_B}$ and $\hat{x}_{B1} > \frac{\alpha_A}{2\gamma_A}$. It shows that the first period equilibrium conflict investment of B will be higher than that of A when B can reach A's revenge peak point but A cannot reach B's revenge peak point. Hence, it implies that B's capability factor is more than that of A, which implies $\frac{\alpha_B}{\gamma_B} > \frac{\alpha_A}{\gamma_A}$.

Observation 4.9. $R_A^3 R_B^{'} > R_B^3 R_A^{'}$ does not hold when $R_A^{'} > 0$ and $R_B^{'} < 0$ i.e., when $\hat{x}_{B1} < \frac{\alpha_A}{2\gamma_A}$ and $\hat{x}_{A1} > \frac{\alpha_B}{2\gamma_B}$. The result $\hat{x}_{B1} > \hat{x}_{A1}$ cannot hold when A can reach B's revenge peak point and B cannot reach A's revenge peak point. It implies that the capability factor of A must be higher than that of B and thus it makes sense that B's (first period) conflict investment cannot be higher than that of A.

Observation 4.10. In short, it is possible when the capability of B is higher than that of A such that B can partially incapacitate A but A cannot partially incapacitate B, then the first period conflict investment of B will be higher than that of A. However, when B cannot partially incapacitate A then for the condition to hold it must be case that A will not be able to partially incapacitate B. That is when $\frac{\alpha_B}{\gamma_B} > \frac{\alpha_A}{\gamma_A}$.

Observation 4.11. The condition for $\hat{x}_{B1} > \hat{x}_{A1}$ also holds when $R_B^{'} > 0$ and $R_A^{'} > 0$, when $R_B^{'} < 0$ and $R_A^{'} < 0$, but looking at the extreme conditions, $\hat{x}_{B1} > \hat{x}_{A1}$ only when capability of B to retaliate is higher than that A i.e., when $\frac{\alpha_B}{\gamma B} > \frac{\alpha_A}{\gamma A}$.

Observation 4.12. Similarly, $\hat{x}_{A1} > \hat{x}_{B1}$ when the capability of A to retaliate is higher than that of B i.e., when $\frac{\alpha_B}{\gamma_B} < \frac{\alpha_A}{\gamma_A}$.

Figure 4.7 interprets equation (4.25), the first period conflict investments of the combatants depending on their capabilities. The 45^o line shows when the first period conflict investments of the combatants are equal ($\hat{x}_{B1} = \hat{x}_{A1}$) i.e., when the 'capabilities' of the

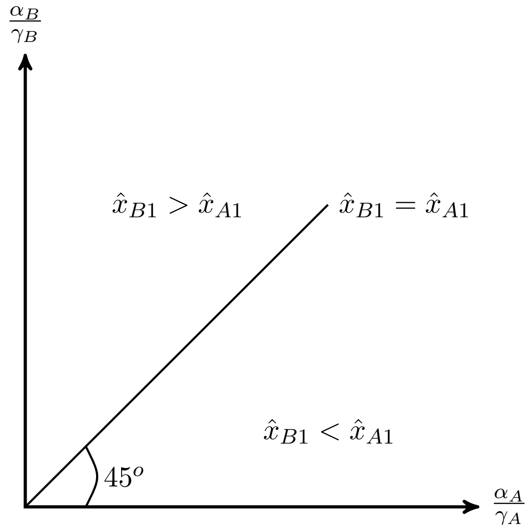

FIGURE 4.7: First period equilibrium conflict investments of the combatants depending on their capabilities to retaliate.

combatants are equal[28]. The figure shows when $\frac{\alpha_B}{\gamma_B} > \frac{\alpha_A}{\gamma_A}$, that is for values above the 45^o line, the first period conflict investment of B is more than that of A and for values below the 45^o line, that is when $\frac{\alpha_B}{\gamma_B} < \frac{\alpha_A}{\gamma_A}$, the first period conflict investment of A is more than that of B.

What happens when both the combatants can partially incapacitate each other? It follows from equations (4.19) and (4.21) and is summarised in the proposition below.

Proposition 4.5. *When $\hat{x}_{A1} > \frac{\alpha_B}{2\gamma_B}$ and $\hat{x}_{B1} > \frac{\alpha_A}{2\gamma_A}$, then the total level of conflict with revenge goes up but the 2nd period level of conflict falls. Thus the intensity of conflict falls with time.*

Observation 4.13. From equation (4.19) we can see that the first period payoff-functions of both the combatants attains maximum at the higher value of conflict effort because of the additional positive term. The positivity of $\frac{d\pi_{i2}}{dx_{i1}}$ can be explained from equation (4.15) and (4.16), both the combatants can incapacitate its opponents to their revenge peak point. Thus, there is no self-deterrence effect in this situation and there is an increase in the first period conflict efforts of both the combatants leading to increase in the level of conflict.

Observation 4.14. But, the level of conflict in the second period falls which is evident from the equation (4.21), since the conflict efforts of both the combatants falls in the

[28]For the sake of simplicity, assuming $\alpha_A = \alpha_B, \gamma_A = \gamma_B$.

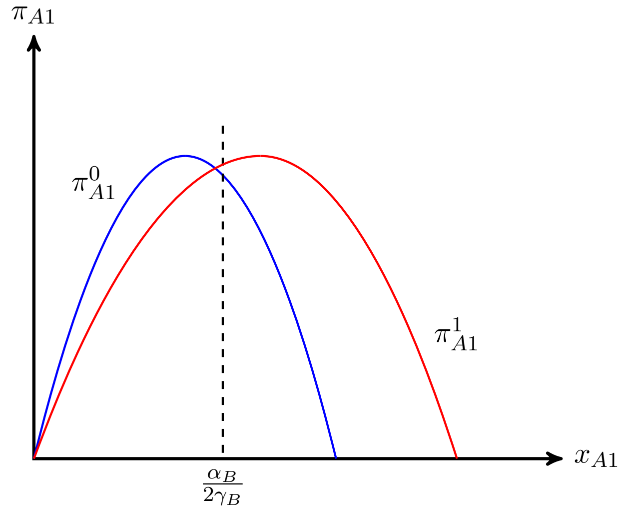

FIGURE 4.8: Payoff function of two different types of combatant A.

2nd period through the incapacitation effect. Thus, the intensity of the conflict starts falling with time[29].

Note: When one combatant is able to almost incapacitate its opponent but the other cannot that is when $\hat{x}_{A1} > \frac{\alpha_B}{2\gamma_B}$ and $\hat{x}_{B1} < \frac{\alpha_A}{2\gamma_A}$, the change in the level of conflict in the presence of revenge is ambiguous. It can be explained in the following way: There are two pressures on the level of conflict in this scenario. An upward pressure from A's increased first period effort as it is capable of incapacitating its opponent, evident from equation (4.19). A's second period conflict effort does not fall because its opponent is not able to reach its revenge peak point. Whereas, there is a downward pressure from B's self-deterrence effect, from equation (4.16) and incapacitation effect from equation (4.21). Thus, B's first period conflict effort falls because of self-deterrence effect and second period conflict effort falls because of incapacitation effect. Thus, the change in the level of conflict in the presence of revenge in this scenario remains ambiguous.

Figure 4.8 shows the first period payoff function w.r.t. first period conflict investment of two different types of combatant A. π^0_{A1} is the first period payoff function of combatant A where the maximisation occurs at a value lower than its opponent's (combatant B) revenge peak point, R^B_P point. Combatant A with this form of payoff function won't be able to incapacitate its opponent to the level that it would dis-incentivize its opponent to go into second period conflict out of revenge, such that its opponent reduces its second period conflict effort. However, π^1_{A1} shows the payoff function of combatant A whose maximisation occurs at a value higher than its opponent's R^B_P point and thus it will be

[29]The Balance of Power model states that when there is equality of power between the conflicting parties there is a chance of peace (Organski and Kugler, 1980).

able to partially incapacitate its opponent and can dis-incentivize B to go into second period conflict out of revenge.

4.7 Narrative Evidence

4.7.1 Japan in World War-II

World War II, also called Second World War is the conflict that involved virtually every part of the world during the years 1939–45. The principal belligerents were the Axis powers; Germany, Italy, and Japan and the Allies, France, Britain, the United States, the Soviet Union, and to some extent China. After an uneasy 20-year pause, World War II was considered a continuation of the disputes left unsettled by World War I[30].

The Empire of Japan entered World War II on 22 September 1940 when it invaded French Indochina, and made its entrance into the war official five days later with the signing of the Tripartite Pact with Germany and Italy on 27 September 1940, though it wasn't until the attack on Pearl Harbor on 7 December 1941 that the U.S. entered the conflict.

The Empire of Japan surrendered and ended World War II, after the United States detonated the first atomic bombs on Hiroshima and Nagasaki and the declaration of war by the Soviet Union[31].

The then emperor of Japan, Hirohito, mentioned about the use of nuclear bombs by U.S. and why surrendering seemed a better option. "Moreover, the enemy has begun to employ a new and most cruel bomb, the power of which to do damage is, indeed, incalculable, taking the toll of many innocent lives. Should we continue to fight, not only would it result in an ultimate collapse and obliteration of the Japanese nation, but also it would lead to the total extinction of human civilization" (Hirohito's 14th August 1945, capitulation announcement[32]). This shows that Japan was completely incapacitated by the United States. U.S reached Japan's incapacitation point ($\frac{\alpha}{\gamma}$) and thus there was no chance that Japan could have carried out a retaliatory attack against the U.S.

[30] https://www.britannica.com/event/World-War-II/Forces-and-resources-of-the-European-combatants-1939

[31] https://www.osti.gov/opennet/manhattan-project-history/Events/1945/surrender.htm

[32] https://www.nytimes.com/1945/08/15/archives/text-of-hirohitos-radio-rescript.html

4.7.2 The 2019-2022 Persian Gulf crisis

The 2019–2022 Persian Gulf crisis is the ongoing state of heightened military tensions between the Islamic Republic of Iran and the United States of America, along with their respective allies, in the Persian Gulf region. Starting in early May 2019, the U.S. began a build-up of its military presence in the region to deter an alleged planned campaign by Iran and its non-state allies to attack American forces and interests in the Persian Gulf and Iraq. This followed a rise in political tensions between the two countries during the Trump administration, which included the withdrawal of the U.S. from the Joint Comprehensive Plan of Action (JCPOA), the imposition of new sanctions against Iran, and the designation of the Islamic Revolutionary Guard Corps (IRGC) as a terrorist organization. In response, Iran designated the United States Central Command as a terrorist organization[33].

In the early 2020 the crisis escalated when a US drone strike in Baghdad killed killed Iranian General Qasem Soleimani. Iran's supreme leader vowed severe revenge on America. "Harsh revenge" awaited the "criminals" who killed Soleimani, Ayatollah Ali Khamenei declared. As per the speculation of the analysts, Iran will try to avoid provoking a full-on war and at the same time it will be a challenge for Iran to retaliate in a way that will domestically save their face without triggering a military response. Iran's military is far less equipped than US and that its economy is collapsing under American sanctions, thus Iran would not want to provoke this war[34]. However, on 8 January 2020 the IRGC launched ballistic missile attacks against two military bases in Iraq housing U.S. soldiers in retaliation for the killing of Soleimani. Although among the facilities struck were troop sleeping quarters, some analysts suggested the strike was deliberately designed to avoid causing any fatalities to dissuade an American response[35]. Now, US's military capability is better than that of Iran. Thus, from Proposition 4.4, we can say that US commits more effort compared to Iran and US is in a favourable position in this conflict with a higher R_P value, which is difficult for Iran to reach.

4.7.3 Israel Palestine conflict

The Israeli-Palestinian conflict dates back to the end of the nineteenth century. On May 14, 1948, the State of Israel was created, leading to the first Arab-Israeli War. The war ended in 1949 with Israel's victory, but 750,000 Palestinians were displaced and the

[33]https://www.bbc.com/news/world-middle-east-49069083 (Accessed on 8/12/2021).

[34]https://www.cnbc.com/amp/2020/01/07/how-iran-could-retaliate-against-the-us-after-solemani-killing.html. (Accessed 15/1/2022).

[35]https://www.theguardian.com/us-news/2020/jan/08/irans-assault-on-us-bases-in-iraq-might-satisfy-both-sides (Accessed on 15/1/2022).

territory was divided into 3 parts: the State of Israel, the West Bank (of the Jordan River), and the Gaza Strip.

Over the years, tensions rose in the region, particularly between Israel and Egypt, Jordan, and Syria. The Camp David Accords improved relations between Israel and its neighbours however the question of Palestinian self-governance remained unresolved. In 1987, thousands of Palestinians living in the West Bank and Gaza Strip rose up against the Israeli government, known as the First Intifada. The 1993 Oslo I Accords mediated the conflict, setting up a framework for the Palestinians to govern themselves in the West Bank and Gaza, and enabled mutual recognition between the newly established Palestinian Authority and Israel's government.

In 2000, the Palestinians infuriated over Israel's control over the West Bank, a stagnating peace process, and former Israeli Prime Minister Ariel Sharon's visit to the Al-Aqsa mosque (the third holiest site in Islam) in September 2000, they launched the Second Intifada, which lasted till 2005[36].

In early October 2023, war broke out between Israel and Hamas[37]. Hamas fired rockets into Israel and stormed into the southern Israeli cities and towns across the border of the Gaza strip, killing and injuring thousands of Israelis[38]. This attack took Israel by surprise and Israel responded quickly with a deadly retaliatory response. The Israel Defence Forces (IDF) launched an unmatched air campaign on the Gaza strip. Israel's aerial campaign involved over 3,000 airstrike events in 85 locations from October to the end of 2023. On 27th October, the IDF with its heavy air power and artillery support moved in hundreds of armoured vehicles and thousands of troops into northern Gaza, surrounding Hamas' stronghold city of Gaza from three sides[39].

This recent Israel-Palestine conflict escalation shows how Israel's military superiority helps it to strongly retaliate against Hamas. Whereas Hamas and its allies continue using asymmetrical strategies against Israeli troops with a motive of imposing higher costs on Israel's forces as they advance into densely populated areas.

The Israel-Palestine conflict can be described as a complicated vicious cycle of violence that does not seem to end. Apart from the demographic, geographic and religious

[36]https://www.cfr.org/global-conflict-tracker/conflict/israeli-palestinian-conflict. (Accessed on 20/12/2021).

[37]Hamas, official name Harakat al-Muqawama al-Islamiya (Islamic Resistance Movement) is a Palestinian Sunni Islamist political and military movement. It was founded by a Palestinian imam and activist Ahmed Yassin in 1987, after the outbreak of the First Intifada against Israel. In 2006, Hamas won the Palestinian legislative election and in 2007 it took control of the Gaza strip from its rival Palestinian party Fatah.

[38]https://www.cfr.org/global-conflict-tracker/conflict/israeli-palestinian-conflict#RecentDevelopments-2 (Accessed on 30/1/2024).

[39]https://acleddata.com/conflict-watchlist-2024/palestine/ (Accessed on 30/1/2024).

complexities, this conflict involves many players (the allies of Israel and Hamas) hence the future of this conflict stays unpredictable. It is still not clear in whose favour the conflict will finally end. However, from a historical perspective and also from the recent violence escalation, we can say from Proposition 4.4, that Israel's strong military capability puts it in a favorable position in this conflict and thus it is able to commit more effort in the conflict compared to Palestine.

4.7.4 2020 India-China skirmishes

Since 5 May 2020, Chinese and Indian troops have engaged in aggressive melee, face-offs and skirmishes at locations along the Sino-Indian border, including near the disputed Pangong Lake in Ladakh and the Tibet Autonomous Region, and near the border between Sikkim and the Tibet Autonomous Region. In late May, Chinese forces objected to Indian road construction in the Galwan River valley. According to Indian sources, melee fighting on 15/16 June 2020 resulted in the deaths of 20 Indian soldiers and casualties of Chinese soldiers not known.

Both India and China are strong, populous, and nuclear-armed countries, with economic ties too, and both the countries have the world's strongest military. China spends a budget of 216 billion dollars on military, ranking third most strongest military in the world and India spends 50 billion dollars on military, ranking fifth most strongest military in the world[40]. As Proposition 4.5 shows, when the factions in the conflict are strong then the intensity of the conflict starts falling with time, this behaviour was also observed in this border conflict. On 25 July 2020, news reports emerged of disengagement at Galwan, Hot Springs and Gogra[41]. Indian authorities (External Affairs Minister S Jaishankar), emphasised that a solution for the recent stand-off between India and China "has to be found in the domain of diplomacy". Hence though tensions are high, it seems that both the countries have resolved to resort to diplomacy to overcome the recent impasse.

Considering the fact that both the countries have similar military capabilities, this situation also corresponds to the SPNE in Figure 4.5, where the combatants are better off without exacting revenge on one another

[40] https://www.businessinsider.in/defense/ranked-the-worlds-20-strongest-militaries/slidelist/51930339.cms. (Accessed on 20/1/2022).

[41] https://www.hindustantimes.com/india-news/india-china-complete-troop-disengagement-at-three-friction-points-focus-now-on-finger-area/story-7aDibG5ICTvksF4R2e0RiN.html (Accessed 20/12/2021).

4.8 Conclusion

This chapter tried to capture the early stages of a conflict that is how much it is probable for a conflict to escalate or de-escalate in the presence of revenge using a simple two-period game of conflict. It discusses in-depth about how and when the value-effect of revenge, self-deterrence effect and the incapacitation effect can influence the strategies of the conflicting parties and thus the intensity of the conflict.

This revenge-capability model of conflict shows that a combatant will engage in a second conflict (out of revenge) only when it has the capability to so given the destruction suffered in the first period conflict. It also shows that it is not always a dominant self-deterrence effect over vengeance effect that leads to de-escalation of conflict but it can also happen because of a more pronounced incapacitation effect. Now, it is mostly assumed that self-deterrence of the combatants is a result of incapacitation of them but this model shows how the self-deterrence effect reduces the first period conflict investment of the combatants and the incapacitation effect reduces the second period conflict investment of the combatants.

We find that when one of the combatant is able to incapacitate its opponent to a certain extent (to its opponent's revenge peak point), its first period conflict effort increases, that is the self-deterrence effect (of the combatant) is not there because that combatant attains a favorable position in the conflict (it is the stronger combatant in the conflict), but the second period conflict investment of the opponent (the weaker combatant) falls because of the incapacitation effect. We also show that when both the combatants are capable enough to partially incapacitate its opponents, then the level of conflict initially goes up (because of increase in first period conflict efforts of both the combatants) but the intensity of the conflict falls with time. Hence, it shows that a conflict can gain momentum in the beginning but as a the conflict continues and the destruction increases it reduces the conflicting parties incentive to further retaliate and thus the intensity of the conflict starts falling with time.

I have briefly discussed about the third-party intervention and impact of the pandemic in this framework in the following section.

4.9 Future extensions of this model

4.9.1 Third-party intervention

Can a third party help its ally in the first period conflict such that its opponent is completely incapacitated?

Suppose, the third party intervenes as an ally of A only in the first period conflict against combatant B for the resource. The third party T provides military subsidies (M) to party A. Using Chang, Potter and Sanders' (2007) cost reduction function, $s = \frac{1}{(1+M)^{\theta}}$, where θ measures the degree of effectiveness with which a dollar of subsidy reduces faction A's unit cost of arming and $0 < \theta < 1$. The third party commits M in period 1 and the third party intervention is exogenous[42].

The first period payoff functions of A and B are;

$$\pi_{A1} = \frac{x_{A1}}{x_{A1} + x_{B1}} V - \frac{1}{(1+M)^{\theta}} x_{A1} + \hat{\pi}_{A2} \tag{4.26}$$

$$\pi_{B1} = \frac{x_{B1}}{x_{A1} + x_{B1}} V - x_{B1} + \hat{\pi}_{B2} \tag{4.27}$$

Maximizing the payoff functions w.r.t x_{i1} where $i = A, B$, The FOCs are;

$$\frac{d\pi_{A1}}{dx_{A1}} = \frac{x_{B1}}{(x_{A1} + x_{B1})^2} V - \frac{1}{(1+M)^{\theta}} + \frac{d\hat{\pi}_{A2}}{dx_{A1}} = 0 \tag{4.28}$$

$$\frac{d\pi_{B1}}{dx_{B1}} = \frac{x_{A1}}{(x_{A1} + x_{B1})^2} V - 1 + \frac{d\hat{\pi}_{B2}}{dx_{B1}} = 0. \tag{4.29}$$

We know that B will be completely incapacitated when $x_{A1} = \frac{\alpha_B}{\gamma_B}$ such that there will be no second period conflict out of revenge[43]. Let's say, the third party helps its ally A with military subsidy such that at equilibrium, $\hat{x}_{A1} = \frac{\alpha_B}{\gamma_B}$.

The first period FOCs when there is no second period conflict out of revenge are;

$$\frac{x_{B1}}{(x_{A1} + x_{B1})^2} V = \frac{1}{(1+M)^{\theta}} \tag{4.30}$$

$$\frac{x_{A1}}{(x_{A1} + x_{B1})^2} = 1 \tag{4.31}$$

Solving for the 1st period equilibrium conflict level of faction B when $\hat{x}_{A1} = \frac{\alpha_B}{\gamma_B}$. From equations (4.30) and (4.31) we get $\hat{x}_{B1} = \frac{\alpha_B}{\gamma_B(1+M)^{\theta})}$

[42] We only want to see the effect of third party intervention in a conflict and not the third party's response to a conflict when there is revenge. However, it remains an avenue to explore in the future

[43] Since $R_B = 0$.

Now, let us calculate the third party intervention function by putting this value of $\hat{x}_{A1}$ and $\hat{x}_{B1}$ in equation (4.30) we get;

$$\frac{\gamma_B(1+M)^\theta V}{\alpha_B(1+(1+M)^\theta)^2} = \frac{1}{(1+M)^\theta} \tag{4.32}$$

$$\sqrt{\frac{\gamma_B V}{\alpha_B}} - 1 = \frac{1}{(1+M)^\theta} \tag{4.33}$$

From, equation (4.33) it is evident that the third-party can help its ally A such that it is able to completely incapacitate its opponent B and prevent a second period conflict out of revenge when $V > \frac{\alpha_B}{\gamma_B}$. This shows when the value of the resource (for which the conflict erupts in the first place) is more than the B's military capability then only the third-party can help its ally A with a military subsidy such that it can incapacitate its opponent B.

4.9.2 Impact of the pandemic

In this case, the pandemic affects the capability part of the revenge function. Let H_i be the amount of resources expended on health infrastructure by country i, thus there is a resource shift from military sector to health sector and this is incorporated in to the revenge function in the following way; $R_i = x_{j1}((\alpha_i - H_i) - \gamma_i x_{j1})$ where, $i, j = A, B$ and $i \neq j$. In this framework full incapacitation of combatant i occurs at $\frac{\alpha_i - H_i}{\gamma_i x_{j1}}$. If the conflict investment of combatant i's opponent is more than the this value then combatant i is fully incapacitated and there won't be any second period conflict out of revenge.

For any country whose priority is the health, there would be a high resource shift and the incentive to retaliate or go into second period conflict would be low because the conflict investment of its opponent has to be be above $\frac{\alpha_i - H_i}{2\gamma_i x_{j1}}$. Thus, when there is a substantial resource shift to the health sector for combatant i then its opponent can incapacitate it to the peak point of its revenge-capability function (R_P^i) for a lower amount of the first period conflict investment.

Chapter 5

Protests during Pandemic: A Game Theoretic Approach

5.1 Introduction

In simple terms, a protest is a public expression of objection and dissent towards an action or idea. Protesters organize protest with a motive to make their opinions heard and influence public opinion or government policy. Throughout history, protests have been a challenge for governments not only in a democratic system but also in a non-democratic and semi-democratic system. When governments face a protest, they a have a choice of either accommodating the protesters or to summon its security apparatus (protest police, riot police) to repress the protesters. History is witness to how repression of protests can lead to violence escalation and radicalization of the public, which can further lead to civil disobedience and this spiral of violence can mark the downfall of the ruling government[1].

Now, suppose there is an external shock like a pandemic, how will the behaviour of the protesters and the government change? When there is a pandemic, organizing a protest and repressing the protesters becomes costly both for the protesters and the government respectively. Thus, it becomes a question that when the intensity of the virus spread changes, when will it be optimal for the protesters to protest in response to a government action and for the government to use a repression strategy. This chapter is an attempt to answer this question, specifically considering the instances from India.

[1]A significant number of empirical literature on protest and collective behavior has found mixed evidence in the context of the effectiveness of government repression in reducing the momentum of a popular protest. For instance, Muller and Weede (1990) found an inverted U relationship, Muller and Opp (1986) found a positive impact. Hibbs (1973) found a negative relationship and Lichbach (1987) and Moore (1998) talked about substitution effects.

On 11 March 2020, the World Health Organization (WHO) declared the novel coronavirus (COVID-19) outbreak a global pandemic. And just like any pandemic the Covid-19 pandemic bought about numerous changes in the existing social and cultural patterns. The Covid-19 pandemic had a significant impact on societies. There were many instances of violence during the pandemic[2]. During the pandemic the world witnessed an upsurge in protests, several civil resistance movements emerged against governmental institutions[3].

With the announcement of the pandemic, demonstrations and protests were initially interrupted since the governments around the world implemented different containment measures like lock-downs, movement restrictions etc., to curb the spread of the virus. Chile experienced a decline in protests[4]. Same temporal pattern was witnessed in India, where a wave of protests and demonstrations began in late 2019 around the Citizenship Amendment Act. Protests declined precipitously following the pandemic declaration in March 2020. Similarly, protests declined in Pakistan after the pandemic declaration. In Nicaragua, protests declined facing the dual threat of virus infection and government repression[5].

The interruption of the protests were short-lived. Initially, the resurgence of the protests came out in the form of direct responses to government handling of the pandemic.[6]. In the year 2020, more than 25 worldwide serious protest movements were directly related to the COVID-19 pandemic (Carothers and O'Donohue, 2020). Shortly thereafter protests evolved into a continuation of the social movements that had begun prior to

[2]Munshi (2021) explored how the COVID-19 pandemic affected world-wide violence both from a macro level and micro level perspective. The author focussed on few of the world conflicts (Yemen, Afghanistan, India-China border conflict) during that time to study the various problems and incentives the conflicting parties were facing under the threat of the COVID-19 pandemic.

[3]Zwet et al., (2022) holistically explored the relationship between the emergence of protests and societal conditions in the context of the COVID-19 pandemic. First, they performed a literature survey to identify the main conditions that lead to the emergence of protests. Then they carried out a quantitative analysis by using statistical and computational modelling to compare the protest dynamics of 27 countries during the pandemic.

[4]On 18th October 2019, Chile witnessed the largest mass protests in its history. The protests were organized by the students against an increase bus fare in Santiago de Chile. The country experienced massive demonstrations and violent clashes between citizens and police leading to a political and social crisis (Cox et al., 2023).

[5]The Armed Conflict Location and Event Data Project (ACLED) has observed the resulting transitions in political violence and protest patterns around the world through their COVID-19 disorder tracker (CDT).

[6]Lange and Monscheuer (2022) explored the impact of large anti-lockdown protests on the spread of coronavirus in Germany. The authors exploited the timing of two large-scale demonstrations in November 2020 and estimated the causal impact of these protests on the spread of coronavirus using an event study framework. They employed novel data on bus connections of travel companies that drove the protesters to these gatherings. Their findings implied significant increase in infection rates in protesters' origin regions after these demonstrations.

the crisis. The economic downturn and the government's mismanagement of the pandemic exacerbated the previously held grievances. In some other cases, new protests and demonstrations emerged altogether (Kishi, 2021).

The introduction of the Citizenship (Amendment) Acton December 11, 2019, sparked massive nationwide protests against the Indian government's decision. On 13 December 2019, the students of Jamia Millia Islamia University undertook a march to the Parliament protesting against the CAA. The police prevented them from going ahead and used batons and tear gas to disperse the protesters leading to clashes with them. Protesters gathered at the Shaheen Bagh road to oppose the Citizenship Amendment Act calling it "anti-Muslim". The protesters also opposed the Delhi Police action against students of Jamia Millia Islamia[7]. Apart from the religious targeting aspect of the act, what attracted massive nationwide and international criticism was the repression of the anti-CAA protests. The Covid-19 emergency imposed severe restrictions on freedom of assembly and peaceful demonstrations. The Shaheen Bagh site was cleared on March 24, 2020 marking the end of the anti-CAA protests[8]. The success of the repression strategy of the government in quelling the momentum of the anti-CAA protests was seen by many as leading to a period of tranquillity during the intensified initial phase of Covid-19 from March to June 2020. This period was characterized by nationwide lockdown and strict social distancing measures.

Following the initial lockdown which lasted till June 2020, subsequent months witnessed a series of protests and nationwide strikes organised by central trade unions and other organisations (Surendran et al., 2024). By September 2020, India found itself amidst a nationwide farmers' uprising against the hasty passage of the farm bills without adequate consultation. The freezing winter followed by the blazing summer and then the repression by the government, did not dissuade protesters from continuing their protests. The police used tear gas and water cannons, used layers of barricades to stop the protesters[9]. Despite being labelled as 'anti-nationals' and 'super-spreaders' the farmers continued with their protests for over a year. With the shadow of the pandemic looming, the farmers' protest led to spike in the virus spread[10]. The pandemic in the backdrop and with the upcoming state elections in the states of Uttar Pradesh and Punjab (majority of the protesting farmers came from these states) in early 2022, the government realised that

[7]https://www.thehindu.com/news/national/police-enter-jamia-millia-islamia-campus-in-delhi-as-anti-citizenship-act-protest-turns-violent/article61606599.ece

[8]https://indianexpress.com/article/cities/delhi/shaheen-bagh-protests-cleared-timeline-caa-delhi-coronavirus-6328911/

[9]https://www.tribuneindia.com/news/haryana/expired-tear-gas-shells-used-in-haryana-to-disperse-punjab-farmers-177431

[10]Parashar et al. (2023) studied the temporal and spatial spread of the Alpha variant (B.1.1.7) in India. They found an unusual spike of Alpha variant-mediated Covid-19 cases in Punjab associated with limited founder events and rapid starlike expansions. The timeline of this event overlapped with the Indian farmers' protest against farm bills.

it cannot afford to keep the protest going thus on 19th November 2021, the government repealed the three acts (Maiorano, 2021).

India witnessed two significant protests during this time period with two different outcomes. This chapter is motivated by these two instances. It attempts to study how the players (government and protesters) will interact in a protest during a pandemic using a signaling game where the protesters' type is imperfectly observed by the government. First, the benchmark case of no-pandemic has been discussed and then it has been compared with the pandemic case. The pandemic is incorporated by using a random variable which captures the virus spread and is common knowledge to both the players. This model shows how the virus spread influences the strategies of the players and the intensity of the protests. We see that at the initial stages of the pandemic the government might prefer to repress the protesters and since the pandemic also increases the cost to protest for the protesters, the protesters might prefer to accept government's proposal and not protest. However, as the pandemic also imposes an additional cost on the government, repression of protests becomes costly for the government. Thus, as virus spread increases the fighting efforts of both the government and the protesters starts falling and when the virus spread increases beyond a certain threshold the government is better-off accommodating the protesters.

5.1.1 Related literature

This chapter uses a theoretical approach to study the strategic behaviour of protesters and the government during the pandemic. There has been limited number of theoretical works in this area. Munshi (2020) for one, theoretically explored the probable effects of a pandemic on conflict by augmenting the Hirshleifer (1991) game theoretic conflict model with externalities. The author examined the different parametric conditions under which a conflict is likely to gain (or lose) momentum when faced with a pandemic. The paper showed that in most of the cases, conflict is likely to fall, while in relatively smaller circumstances, it is likely to rise. Munshi (2019) developed a game-theoretic model to show the interaction between violence and electoral politic. It showed that the more popular political party is likely to allocate more resources to 'extra-electoral' elements, fueling greater violence, an outcome that all voters dislikes.

Pierskalla (2010) developed an extensive strategic game between the government and an opposition group to study the conditions for successful deterrence and protest. The author showed that introducing incomplete information and a third-party threat produces equilibria with repression and exacerbating violence. The model draws implications for the 'murder in the middle' hypothesis[11] (Fein, 1995) and domestic democratic peace.

Another game theoretic exploration of protests is by Ginkel and Smith (1999). They used a game theoretic model with three players, the government, an organized group of dissidents and a mass public to study revolution in repressive regimes. Their model showed that governments seldom offer concessions to protesters and under highly repressive conditions dissident activity is more likely to be successful in influencing large-scale protest. It also confirmed Kuran's (1989) hypothesis, that repressive regimes will collapse suddenly instead of withering away through a gradual loss of legitimacy. The authors used their model to interpret the outcomes that occurred during the successful Velvet Revolution in Czechoslovakia and the failed Tiananmen Square democracy revolution in China.

In the presence of an external shock like the pandemic the payoff information of the players may be perturbed. This is concept of global games. There are quite a few works that have used protest as a signal in a global games setting (Carlsson and van Damme 1993; Morris and Shin 2001). This chapter does not use the concept of global games, it used an extensive game of incomplete information between the government and the protesters to study how an external shock like the pandemic can influence their strategic behaviour. This model shows that at the initial stages of the pandemic the government might prefer to use a repression strategy and the protesters may prefer to not protest. But as the virus spread increases beyond a certain threshold the government will prefer to accommodate the protesters.

The rest of the chapter is organized as follows; section 5.2 discusses the benchmark model of no pandemic; section 5.3 discusses the pandemic model; section 5.5 discusses in details about the anti-CAA protest and the farmers protest in India and which of the signaling equilibria can best explain these protests and section 5.6 concludes.

5.2 The Model - No Pandemic

There is a resource, which is equally valued by both the government and a certain group of people of the country. This resource can be both tangible (like land, water body, mines, etc.) or intangible (giving reservations to certain communities, citizenship's to immigrants, etc.). Let the value of this resource be M (constant). Now, the government proposes an allocation of the resource where $(w_0 M)$ goes to the people and $(1 - w_0)M$ goes to the government, where $0 \leq w_0 < 1$. When that group of people are not satisfied

[11]'More murder in the middle' hypothesis argues that there is a higher level of violation of life-integrity in states that are not fully democratic or free compared to non-democratic or authoritarian states.

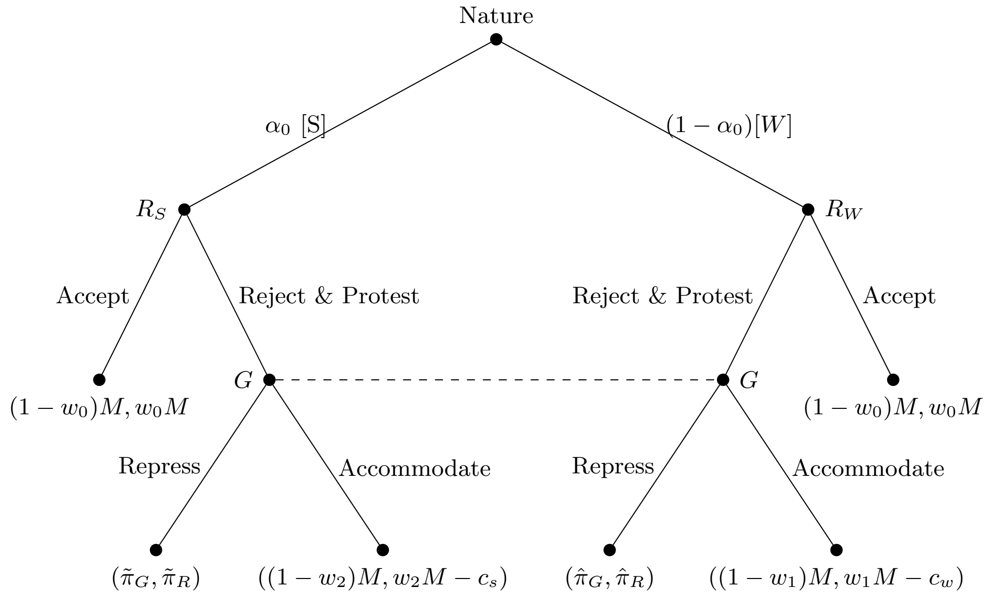

FIGURE 5.1: Game tree: No Pandemic

with the proposed allocation, they may choose to protest. The government can either repress the protesters, which leads to a conflict and the winning side takes control of the entire resource or the government can accommodate the protesters[12]. G knows that it can accommodate the strong type protesters for a higher allocation compared to the weak type protesters, thus, $w_2 > w_1 > w_0$, where, w_2 is the resource allocated to the strong type protesters (R_s) to accommodate them and w_1 to accommodate the weak type protesters (R_w).

The game can be summarised in the following way; there are 2 players; government (G) and protesters (R). G's strength is set and is public knowledge; protesters can be of two types, strong (R_s) and weak (R_w), R knows its type. G does not know R's type. G's prior belief of R being strong type (R_s) is α_0. The cost of protest is higher for the weak type protesters compared to the strong type protesters, $c_w > c_s$ (Figure 5.1).

5.2.1 Strategies of the Players:

The pure strategies of the players, The Protesters (R_s, R_w) and the Government (G) can be written in the following way;

Protester strategy 1: Play 'Reject & Protest' if nature draws R_s and play 'Reject & Protest' if nature draws R_w.

[12]Moore (2000), examined how governments decide to use repression. As per the model the government's decision is based on what the dissident group has just done and not on what it will do next.

Protester strategy 2: Play 'Reject & Protest' if nature draws R_s and play 'Accept' if nature draws R_w.

Protester strategy 3: Play 'Accept' if nature draws R_s and play 'Reject & Protest' if nature draws R_w.

Protester strategy 4: Play 'Accept' if nature draws R_s and play 'Accept' if nature draws R_w.

Government strategy 1: Play 'Repress' if the Protester chooses 'Reject & Protest'.

Government strategy 2: Play 'Accommodate' if the Protester chooses 'Reject & Protest'.

Protester strategy 1 and Protester strategy 4 are called pooling because each type chooses the same action. Protester strategy 2 and Protester strategy 3 are called separating because each type chooses a different action.

5.2.2 Payoffs of the players after repression:

Repression by the government leads to conflict[13]. We calculate the equilibrium conflict investments and equilibrium payoffs of the players, where tilde denotes conflict with strong protesters and hat denotes conflict with the weak protesters.

Let F_i be the 'conflict investment' of player $i, i = G, R_w, R_s$. Let the probability with which player i wins the resource be given by the standard ratio-form Tullock (1980) contest success function written as follows:

$$\pi_R = \frac{\gamma F_R}{\gamma F_R + F_G} M - (F_R + c_i) \tag{5.1}$$

$$\pi_G = \frac{F_G}{\gamma F_R + F_G} M - F_G \tag{5.2}$$

where, γ is the relative conflict-investment effectiveness of the protesters[14]. The payoffs of the players using γ, where $\gamma < 1$ and $c_i = c_w$ for R_w, and $\gamma = 1$ and $c_i = c_s$ for R_s.

[13] A simultaneous game of conflict between the players.

[14] Let b_{Ri} where $i = S, W$, be the per unit conflict investment effectiveness of the protesters and b_G be the per unit conflict investment effectiveness of the Government. $\gamma = \frac{b_{Ri}}{b_G}$ is the relative conflict-investment effectiveness of the protesters. When $\gamma = \frac{b_{Ri}}{b_G} < 1$, it implies that a unit of R_i protester is less effective than a unit official of the government (G).

Solving (5.1) and (5.2) simultaneously when $R = R_w$, we get;

$$\hat{F}_R = \hat{F}_G = \frac{\gamma M}{(1+\gamma)^2} \tag{5.3}$$

$$\hat{\pi}_R = \frac{\gamma^2 M}{(1+\gamma)^2} - c_w \quad , \quad \hat{\pi}_G = \frac{M}{(1+\gamma)^2} \tag{5.4}$$

The payoffs of the players when $R = R_s$, $\gamma = 1$.

$$\tilde{F}_R = \tilde{F}_G = \frac{M}{4} \tag{5.5}$$

$$\tilde{\pi}_R = \frac{M}{4} - c_s \quad , \quad \tilde{\pi}_G = \frac{M}{4} \tag{5.6}$$

Observation 5.1. G has to expend more effort with the strong protester, $\tilde{F}_G > \hat{F}_G$. The strong protesters expends more effort than the weak protesters, $\tilde{F}_R > \hat{F}_R$.

Observation 5.2. The expected payoff of the government is higher when repressing the weak protesters compared to the strong protester, $\tilde{\pi}_G < \hat{\pi}_G$. The expected payoff of the strong protesters is greater than that of the weak protesters, $\tilde{\pi}_R > \hat{\pi}_R$.

The intensity of the protest (X_i) is the standard sum of the fighting efforts of the players.

$$X_S = \tilde{F}_G + \tilde{F}_R = \frac{M}{2} \tag{5.7}$$

$$X_W = \hat{F}_G + \hat{F}_R = \frac{2\gamma M}{(1+\gamma)^2} \tag{5.8}$$

The intensity of the protest is greater when the protesters are strong, $X_S > X_W$.

5.2.3 Equilibrium: No Pandemic

The game has been solved using the concept of perfect Bayesian equilibrium (PBE). The analysis is confined to pure strategy PBEs.

The game has two classes of PBEs depending on the levels of proposed allocation and payoffs after repression and accommodation. The first class of PBE is a separating PBE where one of the type of protester rejects the government's proposed allocation ($w_0 M$) and protests and the other type accepts the government's proposed allocation ($w_0 M$). The government then decides whether to repress or accommodate the protesters depending on the type. The second class of PBE is a pooling PBE where both the types of protesters either accepts the government's proposed allocation or both the types rejects the government's proposed allocation and protest. The government decides whether to repress or accommodate the protesters depending on the prior belief of whether the protesters are strong or not.

Proposition 5.1. *The separating PBEs where the government (G) believes that if there is a protest it is by the strong protesters (R_s) will only hold.*

Proof. Government's (G) beliefs over the 'Reject & Protest' information set $[R_s, R_w] = \{1, 0\}$. G believes that the strong protesters (R_s) will reject the proposed allocation and protest and the weak protesters (R_w) will accept the proposed allocation.

Separating equilibrium A: The separating PBE where the strong protesters (R_s) will reject the proposed allocation and protest and the weak protesters (R_w) will accept the proposed allocation and the government (G) will accommodate the protesters will hold when[15];

$$w_2 < \frac{3}{4} \tag{5.9}$$

$$w_1 - \frac{c_w}{M} < w_0 < w_2 - \frac{c_s}{M} \tag{5.10}$$

Thus, the separating PBE when (5.9) and (5.10) holds is; Strong protester (R_s) will 'Reject & Protest' and Weak Protester (R_w) will 'Accept'. Government (G) with belief $\{1, 0\}$ over the information set $[R_s, R_w]$ will 'Accommodate'.

Separating equilibrium B: The separating PBE where the strong protesters (R_s) will reject the proposed allocation and protest and the weak protesters (R_w) will accept the proposed allocation and the government will repress the protesters will hold when[16];

$$w_2 > \frac{3}{4} \tag{5.11}$$

$$\frac{\gamma^2}{(1+\gamma)^2} - \frac{c_w}{M} < w_0 < \frac{1}{4} - \frac{c_s}{M} \tag{5.12}$$

Thus, the separating PBE when (5.11) and (5.12) holds is; Strong protester (R_s) will 'Reject & Protest' and Weak Protester (R_w) will 'Accept'. Government (G) with belief $\{1, 0\}$ over the information set $[R_s, R_w]$ will 'Repress'.

[15]Complete proof shown in Appendix C.0.1.
[16]Complete proof shown in Appendix C.0.2.

Now, let us consider the other case;

Government (G) believes that the weak protesters (R_w) will reject the proposed allocation and protest and the strong protesters (R_s) will accept the proposed allocation i.e., G's beliefs over the 'Reject & Protest' information set $[R_s, R_w] = \{0, 1\}$.

The separating PBE where the weak protesters (R_w) will reject the proposed allocation and protest and the strong protesters (R_s) will accept the proposed allocation and the government (G) will accommodate the protesters will not hold[17].

The separating PBE where the weak protesters (R_w) will reject the proposed allocation and protest and the strong protesters (R_s) will accept the proposed allocation and the government (G) will repress the protesters will not hold[18].

The government (G) accommodates the strong protesters for a higher proportion compared to the weak protesters. The cost of protest is also higher for the weak protester and under repression the expected payoff of the strong protesters is greater than that of the weak protesters. Thus, given G's actions it is optimal for the government to believe that if there is a protest, it is carried out by the strong protesters.

■

Proposition 5.2. *The pooling PBE where both the type of protesters (R_s, R_w) will reject the government's (G) proposed allocation and protest and the government (G) will repress (or accommodate) the protesters when;*

i) The proposed allocation $(w_0 M)$ is less than the payoffs of both the types of protesters when they protest and the cost of protest (c_s, c_w) is also negligible for both the types, they will accept the proposed allocation otherwise.

ii) The government (G) will repress the protesters when the relative probability of the protesters being the strong type is less, i.e, when $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\tilde{\pi}_G-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G}$ and will accommodate otherwise.

Proof. The pooling equilibria can be written in the following way;

Pooling equilibrium A:
Strong Protesters (R_s) and Weak Protesters (R_w): Reject & Protest

[17] Complete proof shown in Appendix C.0.3.
[18] Complete proof shown in Appendix C.0.4.

Government (G): Repress

The government G will repress in the 'Reject & Protest' information set when $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\hat{\pi}_G-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G}$. That is when the relative probability of the protesters being strong is less the government's best response will to be repress the protesters.

Both the strong and the weak type of protesters (R_s, R_w) will reject the proposed allocation and protest when the payoff from protest under repression is more than the proposed allocation. Thus, $\tilde{\pi}_R > \hat{\pi}_R > w_0 M$.

Pooling equilibrium B:

Strong Protesters (R_s) and Weak Protesters (R_w): Reject & Protest

Government (G): Accommodate

The government G will accommodate in the 'Reject & Protest' information set when $\frac{\alpha_0}{(1-\alpha_0)} > \frac{\hat{\pi}_G-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G}$. That is when the relative probability of the protesters being strong is more the government's best response will to be accommodate the protesters.

Both the strong and the weak type of protesters (R_s, R_w) will reject the proposed allocation and protest when the payoff from protest under accommodation is more than the proposed allocation., $w_2 M - c_s > w_1 M - c_w > w_0 M$.

Pooling Equilibrium C:

Strong Protesters (R_s) and Weak Protesters (R_w): Accept

Government (G): Repress

Similarly, like Pooling equilibrium A, the government G will repress in the 'Reject & Protest' information set when $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\hat{\pi}_G-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G}$.

Both the strong and the weak type of protesters (R_s, R_w) will accept the proposed allocation when the payoff from protest under repression is less than the proposed allocation. Thus, $w_0 M > \tilde{\pi}_R > \hat{\pi}_R$.

Pooling Equilibrium D:

Strong Protesters (R_s) and Weak Protesters (R_w): Accept

Government (G): Accommodate

Similarly, like Pooling equilibrium B, the government G will accommodate in the 'Reject & Protest' information set when $\frac{\alpha_0}{(1-\alpha_0)} > \frac{\hat{\pi}_G-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G}$.

Both the strong and the weak type of protesters (R_s, R_w) will accept the proposed allocation when the payoff from protest under accommodation is less than the proposed allocation., $w_0 M > w_2 M - c_s > w_1 M - c_w$.

■

5.2.4 Equilibrium Refinements:

We will check which of the PBEs will sustain under the Cho-Kreps (1987) intuitive criteria.

The Cho-Kreps (1987) intuitive criterion is used to evaluate Perfect Bayesian Equilibrium (PBE), it basically rules out PBEs based on unrealistic beliefs[19]. Let us check which equilibria sustains under the intuitive criterion in this model.

Let us consider the Pooling equilibrium C, where both the types of protesters accept the proposed allocation (w_0), and the government (G) represses in the 'Reject & Protest' information set.

Considering the case where the government's (G) best response is to accommodate the strong protesters, $\tilde{\pi}_G < (1-w_2)M$, and its best response is to repress the weak protesters $\hat{\pi}_G > (1-w_1)M$. G will repress in the in the 'Reject & Protest' information set when $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\hat{\pi}_G-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G}$ i.e., the relative probability of the protesters being strong is less. Both the protesters will accept (and not deviate) when $w_0 M > \tilde{\pi}_R > \hat{\pi}_R$.

Now, suppose the strong protesters (R_s) are better-off under accommodation compared to the equilibrium payoff from 'Accept', ($w_2 M - c_s > w_0 M$). The weak-protesters (R_w) are worse-off under repression and accommodation if they protest compared to their equilibrium payoff. Thus, for the weak protesters (R_w) the equilibrium payoff is strictly dominating. In this situation, if the government finds itself in the off-the-equilibrium path i.e., if there is protest, G will know it is the strong protesters and G will be better-off accommodating the strong protesters and the strong protesters will know it and they will protest. Thus, this equilibrium will not sustain under the intuitive criteria.

In short, Pooling equilibrium C will not sustain when the cost of protest for the strong protesters (R_s) is such that, $(\frac{1}{4} - w_o)M < c_s < (w_2 - w_o)M$ and the allocation after accommodation is such that $\frac{1}{4} < w_2 < \frac{3}{4}$. For the weak protesters (R_w) the cost of protest is high such that $c_w > (w_1 - w_o)M$ and the allocation after accommodation is such that $w_1 > 1 - \frac{1}{(1+\gamma)^2}$.

If we consider the case, where the equilibrium-payoff from acceptance is dominating for the strong protesters but it is not so for the weak protesters. Now, this is not possible,

[19] A PBE fails to sustain under the intuitive criterion when;

- Some type t of player 1 can be strictly better-off by deviating (relative to the equilibrium payoff) if other players believe it to be type t under the deviation.
- No other type t' can be strictly better-off under any beliefs by making the same deviation as type t.

because we already know that the payoff for the strong protesters is greater than the weak protesters both under repression and accommodation.

However, if we consider Pooling equilibrium D where the government accommodates in the 'Reject & Protest' information set, and the protesters accept. This PBE will always sustain under intuitive criteria[20]. It shows that if the protesters are better-off in accepting the proposed allocation even when the government is willing to accommodate them then the equilibrium payoff from acceptance is dominating for both the types.

To summarise, Pooling equilibrium C may or may not sustain under certain conditions whereas Pooling equilibrium D will always sustain.

5.3 The Model - With Pandemic

Now let us incorporate the pandemic in this protest scenario. There is a pandemic and a random variable $v \sim f(v)$ captures the virus spread. Due to the pandemic, G's cost of fighting the conflict rises, it adds an additional cost of fighting the conflict which depends on the virus spread. G's signal of the virus spread remains same irrespective of the type of the opponent, i.e., $f(v|S) = f(v|W)$.

Cost of protest for the players:

The pandemic increases the cost of protest for the players. The cost of protest for the players are:
The strong protesters (R_s): $c_s^P > c_s$;
The weak protesters (R_w): $c_w^P > c_w$;
Like the no-pandemic scenario, the cost of protest is higher for the weak type protesters compared to the strong type protesters: $c_w^P > c_s^P$.

The government's (G) cost of conflict (with pandemic) becomes $(F_G + h(v)F_G) = (1 + h(v))F_G$.
Let $k(v) = (1 + h(v))$ where $h(v)$ captures the additional cost of fighting the conflict due to the pandemic, $h'(v) > 0, h''(v) \geqq 0$.
This cost function shows how the cost of repression becomes sensitive to the virus spread.

[20]Proof in Appendix C.0.5.

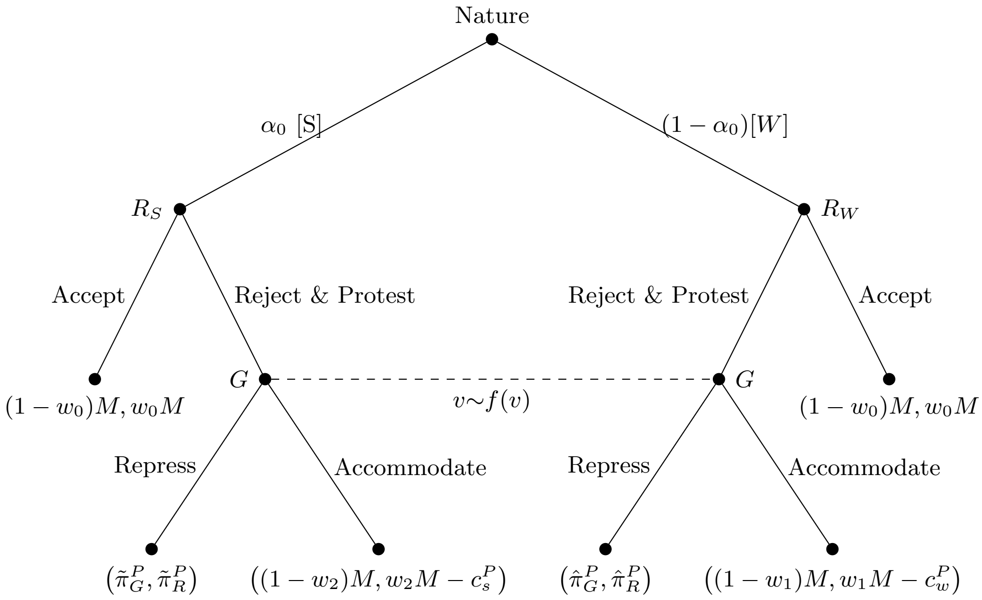

FIGURE 5.2: Game tree: Pandemic

5.3.1 Strategies of the Players:

The pure strategies of the players, The Protesters (R_s, R_w) and the Government (G) is same as it was in no-pandemic scenario (Figure 5.2).

5.3.2 The payoffs of the players after repression:

Similarly, like the no-pandemic case repression leads to conflict. The payoffs of the players are given below;

$$\pi_R = \frac{\gamma F_R}{\gamma F_R + F_G} M - (F_R + c_w^P) \tag{5.13}$$

$$\pi_G = \frac{F_G}{\gamma F_R + F_G} M - k(v) F_G \tag{5.14}$$

where, γ is the relative conflict-investment effectiveness of the protesters and $\gamma < 1$ for R_w and $\gamma = 1$ for R_s.

Solving (5.13) and (5.14) simultaneously we get;

$$\hat{F}_R^P = \frac{k(v)\gamma M}{(1+\gamma k(v))^2} \tag{5.15}$$

$$\hat{F}_G^P = \frac{\gamma M}{(1+\gamma k(v))^2} \tag{5.16}$$

$$\hat{\pi}_R^P = \frac{(k(v)\gamma)^2 M}{(1+k(v)\gamma)^2} - c_w^P \quad , \quad \hat{\pi}_G^P = \frac{M}{(1+k(v)\gamma)^2} \tag{5.17}$$

The payoffs of the players when $R = R_s, \gamma = 1$;

$$\tilde{F}_R^P = \frac{k(v)M}{(1+k(v))^2} \tag{5.18}$$

$$\tilde{F}_G^P = \frac{M}{(1+k(v))^2} \tag{5.19}$$

$$\tilde{\pi}_R^P = \frac{k(v)^2 M}{(1+k(v))^2} - c_s^P \quad , \quad \tilde{\pi}_G^P = \frac{M}{(1+k(v))^2} \tag{5.20}$$

Observation 5.3. Similarly, in the pandemic situation, the government G is better-off repressing the weak protesters (R_w) compared to the strong protester (R_s) i.e., $\tilde{\pi}_G^P < \hat{\pi}_G^P$. The expected payoff of the strong protesters (R_s) under repression is more than that of the weak protesters (R_w) i.e., $\tilde{\pi}_R^P > \hat{\pi}_R^P$.

The intensity of the protest with-pandemic (X_i^P) is given below;

$$X_s^P = \tilde{F}_G^P + \tilde{F}_R^P = \frac{M}{(1+k(v))} \tag{5.21}$$

$$X_w^P = \hat{F}_G^P + \hat{F}_R^P = \frac{\gamma M(1+k(v))}{(1+\gamma k(v))^2} \tag{5.22}$$

Proposition 5.3. *1) The fighting effort of the government falls during the pandemic, both with the strong protesters and weak protesters, i.e.,* $\tilde{F}_G > \tilde{F}_G^P$ *and* $\hat{F}_G > \hat{F}_G^P$*.*

2) The fighting effort of the strong protesters falls during the pandemic, i.e, $\tilde{F}_R > \tilde{F}_R^P$*. However, the fighting effort of the weak protesters may rise or fall depending on the virus spread such that* $\hat{F}_R > \hat{F}_R^P$ *when* $k(v) > \frac{1}{\gamma^2}$ *and* $\hat{F}_R < \hat{F}_R^P$ *when* $k(v) < \frac{1}{\gamma^2}$*.*

In the no-pandemic scenario, the equilibrium fighting effort of the government and the protesters (both the types) were equal. However, in the post-pandemic scenario the equilibrium fighting effort of the government is lower than the equilibrium fighting effort of the protesters (both the types), owing to its increase in the cost of conflict. The fighting effort of both the government and strong protesters falls when there is a pandemic. Thus, the protest intensity falls when there is a pandemic.

The fighting effort of the weak protesters might rise or fall depending on the virus spread and the conflict investment effectiveness of the government relative to the protesters. When the virus spread is higher than the conflict investment efficiency of the government relative to the protesters, the government will lower its conflict-investment (substantially) and at equilibrium the protesters will also lower its conflict investment. However, if the virus spread is such that it is lower than the relative conflict-investment efficiency of the government then the fighting effort of the weak protesters post pandemic will rise compared to its no-pandemic fighting effort. The protest intensity post pandemic might rise or fall compared to the no-pandemic depending on the level of the virus spread and the relative conflict-investment effectiveness of the protesters. In short, the intensity of protests starts falling with the virus spread.

Best response of G:

The government's (G) strategies and actions will now depend on the level of virus spread. Let us first check when the government (G) will accommodate/repress in the 'Reject & Protest' information set.

Let us check the best response of the government (G) under different levels of virus spread.

1) G will be better-off accommodating the strong protester (R_s), when;

$$\tilde{\pi}_G^P = \frac{M}{(1+k(v))^2} < (1-w_2)M \tag{5.23}$$

$$\frac{1}{\sqrt{(1-w_2)}} - 2 < h(v) \tag{5.24}$$

$$\text{Let, } \; h(v_s^*) = \frac{1}{\sqrt{(1-w_2)}} - 2 \tag{5.25}$$

When $h(v) > h(v_s^*)$, G will accommodate R_s otherwise repress when $h(v) < h(v_s^*)$.

2) G will be better-off accommodating the weak protester (R_w) when;

$$\hat{\pi}_G^P = \frac{M}{(1+\gamma k(v))^2} < (1-w_1)M \tag{5.26}$$

$$\frac{1}{\gamma}(\frac{1}{\sqrt{(1-w_1)}} - 1) - 1 < h(v) \tag{5.27}$$

$$\text{Let,}\ \ h(v_w^*) = \frac{1}{\gamma}(\frac{1}{\sqrt{(1-w_1)}} - 1) - 1 \tag{5.28}$$

When $h(v) > h(v_w^*)$, G will accommodate R_w otherwise repress when $h(v) < h(v_w^*)$.

Since $h(v)$ is an increasing function of v hence, $v > v_i^* \implies h(v) > h(v_i^*)$

The best responses of G in the 'Reject & Protest' information set can be written in the following way:

1. G will Repress R_i iff $h(v) < h(v_i^*)$ where $i = s, w$.
2. G will Accommodate R_i iff $h(v) > h(v_i^*)$ where $i = s, w$.
3. G will Repress both R_s, R_w iff $h(v) < \min\{h(v_s^*), h(v_w^*)\}$.
4. G will Accommodate both R_s, R_w iff $h(v) > \max\{h(v_s^*), h(v_w^*)\}$.

Let us check, when $h(v_s^*) > h(v_w^*)$ and when $h(v_s^*) < h(v_w^*)$;

Subtracting (5.25) from (5.28);

$$h(v_s^*) - h(v_w^*) = \left(\frac{1}{\sqrt{(1-w_2)}} - 2\right) - \left(\frac{1}{\gamma}\left(\frac{1}{\sqrt{(1-w_1)}} - 1\right) - 1\right) \tag{5.29}$$

$$h(v_s^*) - h(v_w^*) = \frac{1}{\gamma}\left(\frac{\gamma(1-\sqrt{1-w_2})}{\sqrt{1-w_2}} - \frac{(1-\sqrt{1-w_1})}{\sqrt{1-w_1}}\right) \tag{5.30}$$

$$h(v_s^*) \lesseqgtr h(v_w^*)\ \ \text{when}\ \ \gamma \lesseqgtr \left(\frac{1-\sqrt{1-w_1}}{1-\sqrt{1-w_2}}\right)\left(\frac{\sqrt{1-w_2}}{\sqrt{1-w_1}}\right) \tag{5.31}$$

$$\frac{1-\sqrt{1-w_1}}{1-\sqrt{1-w_2}} < 1\ \ \text{and}\ \ \frac{\sqrt{1-w_2}}{\sqrt{1-w_1}} < 1 \tag{5.32}$$

Proposition 5.4. *When γ is greater than the relative value of what G gets when R is strong (versus when R is weak), the virus threshold of accommodating R_s is greater than that of the R_w. Thus, if γ converges to 1 (technology of fighting of R_s and R_w are* *almost equal), then G fights R_s to a higher virus threshold compared to R_w, since G has to accommodate the R_s for a greater proportion.*

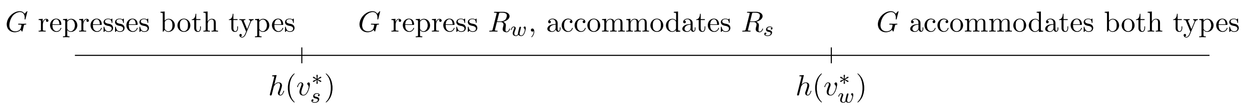

FIGURE 5.3: $h(v_s^*) < h(v_w^*)$ when $\gamma < \left(\frac{1-\sqrt{1-w_1}}{1-\sqrt{1-w_2}}\right)\left(\frac{\sqrt{1-w_2}}{\sqrt{1-w_1}}\right)$

Government's (G) actions depend on the level of the virus spread. It shows how the government will prefer to repress in the initial stages of the pandemic. It is something we observed with the anti-CAA protests in India. The government went on repressing till the protest was abandoned by the protesters. The government (G) will prefer to accommodate the protesters when the virus spread is above a certain a level. Considering the farmers' protest in India when the country was in the midst of the second wave of the Covid-19 pandemic, the government tried to accommodate the farmers by the suspending the implementation of the law for 18 months[21]. This offer was refused by the farmers and they continued with the protest. The cost function shows how the cost of conflict for the government becomes sensitive to level of virus spread. In this case, the state elections in Punjab and Uttar Pradesh (majority of the protesting farmers came from there) in early 2022 made the cost of repression for the government more sensitive to the virus spread. With the pandemic and the upcoming state elections the government was left with no alternatives but to repeal the laws.

Numerical Example:

Let us use a numerical example to explain the model.

Let $v \sim f(v)$ be a uniform distribution over $[\underline{v}, \overline{v}]$, where $f(v) = \frac{1}{\overline{v}-\underline{v}}$, $\underline{v}, \overline{v} > 0$.

$h(v)$ is the additional cost of fighting the conflict, where $h(v) = N \int_{\underline{v}}^{v} v f(v) dv$. N is the total population. The additional cost of conflict, $h(v)$ is the expected number of population getting virus infected.

[21]In January 2021, Union Minister for Agriculture, Narendra Singh Tomar, offered to halt the implementation of the law for 18 months. https://www.livemint.com/news/india/govt-ready-to-suspend-farm-laws-for-18-months-farmers-to-consider-proposal-tomorrow-11611151703815.html

$$h(v) = N\int_{\underline{v}}^{v} \frac{v}{\overline{v}-\underline{v}} dv \tag{5.33}$$

$$h(v) = \frac{N(v^2-\underline{v}^2)}{2(\overline{v}-\underline{v})} \tag{5.34}$$

Let us simplify and assume $\underline{v} = 0$, $\overline{v} = 1$ and the total population N be a numeraire, $N = 1$. So $h(v) = \frac{v^2}{2}$. The virus thresholds are;

$$v_s^* = \sqrt{2\left(\frac{1}{\sqrt{(1-w_2)}} - 2\right)} \geq 0, \quad when \quad w_2 > \frac{3}{4} \tag{5.35}$$

$$v_w^* = \sqrt{\left(\frac{1}{\gamma}\left(\frac{1}{\sqrt{(1-w_1)}} - 1\right) - 1\right)} \geq 0, \quad when \;\; w_1 > 1 - \frac{1}{(1+\gamma)^2} \tag{5.36}$$

The fighting efforts of the players are:

$$\tilde{F}_R^P = \frac{2M(4+v^2)}{(2+v^2)^2} \tag{5.37}$$

$$\tilde{F}_G^P = \frac{4M}{(4+v^2)^2} \tag{5.38}$$

$$\hat{F}_R^P = \frac{2\gamma M(2+v^2)}{(2(1+\gamma)+v^2)^2} \tag{5.39}$$

$$\hat{F}_G^P = \frac{4\gamma M}{(2(1+\gamma)+v^2)^2} \tag{5.40}$$

Let us check the no-pandemic and pandemic differences.

The intensity of the protest is the standard sum of the fighting efforts of the players.

$X_S = \tilde{F}_G + \tilde{F}_R$ and $X_W = \hat{F}_G + \hat{F}_R$.
$X_S^P = \tilde{F}_G^P + \tilde{F}_R^P$ and $X_W^P = \hat{F}_G^P + \hat{F}_R^P$.

Now, $X_s > X_s^P$. When the protesters are strong, the conflict intensity in the presence of pandemic is less than the no-pandemic conflict intensity. This holds for all types of distribution. For this distribution, $X_w > X_w^P$, at all levels of virus spread. Since G's cost of conflict rises, it reduces its conflict efforts and thus at equilibrium the protesters reduces its conflict efforts. The equation below shows that the conflict effort of the protesters fall with the rising virus spread.

$$\frac{d\tilde{F}_R^P}{dv} < 0, \quad since \;\; k'(v) > 0 \;\; and \;\; k(v) > 1 \tag{5.41}$$

$$\frac{d\hat{F}_R^P}{dv} < 0, \quad since \;\; k'(v) > 0 \;\; and \;\; k(v) > 1 \tag{5.42}$$

The payoffs of the players are:

$$\tilde{\pi}_R^P = \frac{M(2+v^2)^2}{(4+v^2)^2} - c_s^P \tag{5.43}$$

$$\tilde{\pi}_G^P = \frac{4M}{(4+v^2)^2} \tag{5.44}$$

$$\hat{\pi}_R^P = \frac{\gamma^2 M(2+v^2)^2}{(2(1+\gamma)+v^2)^2} - c_w^P \tag{5.45}$$

$$\hat{\pi}_G^P = \frac{4M}{(2(1+\gamma)+v^2)^2} \tag{5.46}$$

The expected conflict- payoff of the government (G) falls with the level of virus spread, $\frac{d\tilde{\pi}_G^P}{dv} < 0$ and $\frac{d\hat{\pi}_G^P}{dv} < 0$. However, the expected conflict payoffs of the protesters rises with the level of virus spread, $\frac{d\tilde{\pi}_R^P}{dv} > 0$ and $\frac{d\hat{\pi}_R^P}{dv} > 0$.

5.4 Equilibrium - With Pandemic:

In this section we will discuss about the PBEs in the presence of the pandemic. Like the no-pandemic case, there are two classes of PBEs, the first class of PBE is the separating PBE and the second class of PBE is the pooling PBE.

Now, there will be pooling and separating equilibria for each level of virus spread. We will discuss the equilibria under different levels of virus spread given $\gamma < \left(\frac{1-\sqrt{1-w_1}}{1-\sqrt{1-w_2}}\right)\left(\frac{\sqrt{1-w_2}}{\sqrt{1-w_1}}\right)$;

Proposition 5.5. *Like the no-pandemic case, in the pandemic scenario the separating PBE where the government's belief is such that if there is a protest it is by the strong protesters will only hold.*

The reason being that the payoff for the strong type of protesters (R_s) is more than that of the weak type (R_w) both under repression and accommodation so if the weak-type's (R_w) best response is to 'Reject and protest' then the strong-type's (R_s) best-response must be 'Reject and protest'. Thus, if there is protest government (G) will always believe it is by the strong type.

The pooling and separating equilibria has been discussed in the following subsections.

5.4.1 When $h(v) < h(v_s^*)$, G will 'Repress' in the 'Reject & Protest' information set

Pooling equilibrium P.1:
Strong Protesters (R_s) and Weak Protesters: Reject & Protest
Both the strong and the weak type of protesters (R_s, R_w) will reject the proposed allocation and protest when the payoff from protest under repression is more than the proposed allocation. Given $h(v) < h(v_s^*)$, this PBE where both types of protesters will protest and the government will repress will hold when $\tilde{\pi}_R^P > \hat{\pi}_R^P > w_0 M$.

Pooling equilibrium P.2:
Strong Protesters (R_s) and Weak Protesters: Accept
Both the strong and the weak type of protesters (R_s, R_w) will accept the proposed allocation when the payoff from protest under repression is less than the proposed allocation. Given $h(v) < h(v_s^*)$, this PBE where both types of protesters accepts the proposed allocation and the government represses will hold when, $w_0 M > \tilde{\pi}_R^P > \hat{\pi}_R^P$.

Separating Equilibrium S.1:
Strong protesters (R_s): Reject & Protest.
Weak protesters (R_w): Accept.
Government(G): Beliefs in the protest information set $[R_s, R_w] = \{1, 0\}$, G believe the strong protesters (R_s) protest. The government (G) represses in the 'Reject & Protest' information set. This PBE holds when $\hat{\pi}_R^P < w_0 M < \tilde{\pi}_R^P$. That is, when $\frac{(k(v)\gamma)^2 M}{(1+k(v)\gamma)^2} - c_w^P < w_0 M < \frac{k(v)^2 M}{(1+k(v))^2} - c_s^P$.

Considering the other case where the weak protesters R_w reject the proposed allocation and protest and the strong protesters R_s accept the proposed allocation. The government's (G) belief in the 'Reject & Protest' information set, $[R_s, R_w] = \{0, 1\}$, G believes that the weak protesters (R_w) protest. This PBE will hold when $\tilde{\pi}_R^P < w_0 M < \hat{\pi}_R^P$. That is when $\frac{k(v)^2 M}{(1+k(v))^2} - c_s^P < w_0 M < \frac{(k(v)\gamma)^2 M}{(1+k(v)\gamma)^2} - c_w^P$. This is not possible since $\frac{k(v)^2 M}{(1+k(v))^2} - c_s^P > \frac{(k(v)\gamma)^2 M}{(1+k(v)\gamma)^2} - c_w^P$. Thus, this PBE will not hold.

5.4.2 When $h(v) > h_w(v)^*$, G will 'Accommodate' in the 'Reject & Protest' information set.

Pooling equilibrium P.3:
Strong Protesters (R_s) and Weak Protesters: Reject & Protest

Both the strong and the weak type of protesters (R_s, R_w) will reject the proposed allocation and protest when the payoff from protest under accommodation is more than the proposed allocation. Given $h(v) > h_w(v)^*$, this PBE where both types of protesters will protest and the government will accommodate them will hold when $(w_2M - c_s^P) > (w_1M - c_w^P) > w_0M$.

Pooling equilibrium P.4:
Strong Protesters (R_s) and Weak Protesters: Accept
Both the strong and the weak type of protesters (R_s, R_w) will accept the proposed allocation when the payoff from protest under accommodation is less than the proposed allocation. Given $h(v) > h_w(v)^*$, this PBE where both types of protester accept the proposed allocation and the government accommodates will hold when $w_0M > (w_2M - c_s^P) > (w_1M - c_w^P)$.

Separating Equilibrium S.2:
Strong protesters (R_s): Reject & Protest
Weak protesters (R_w): Accept
Government(G): Beliefs in the protest information set $[R_s, R_w] = \{1, 0\}$, G believe the strong protesters (R_s) protest. The government (G) accommodates the protesters in the 'Reject & Protest' information set. G believes that weak protesters (R_w) do not protest and the game ends when there is no protest. This PBE holds when $(w_1M - c_w^P) < w_0M < (w_2M - c_s^P)$.

Similarly, the other case (of PBE) where the weak protesters R_w reject the proposed allocation and protest and the strong protesters R_s accept the proposed allocation and the government G believes that the weak protesters (R_w) protest, will not hold. Since, $(w_1M - c_w^P) < (w_2M - c_s^P)$.

5.4.3 When $h_s(v)^* < h(v) < h_w(v)^*$, G's best-response is to 'Repress' R_w and 'Accommodate' R_s.

Pooling equilibrium P.5:
Strong Protesters (R_s) and Weak Protesters (R_w): Reject & Protest
Government (G): Repress
G represses when there is protest when $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$.
Both the strong and the weak type of protesters (R_s, R_w) will reject the proposed allocation and protest when the payoff from protest under repression is more than the proposed allocation. Given belief, $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$, this PBE where both the types of protesters will protest and the government will repress will hold when, $\tilde{\pi}_R^P > \hat{\pi}_R^P > w_0M$.

Pooling equilibrium P.6:

Strong Protesters (R_s) and Weak Protesters: Accept

Government (G): Repress

G represses when there is protest when $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$.

Both the strong and the weak type of protesters (R_s, R_w) will accept the proposed allocation when the payoff from protest under repression is less than the proposed allocation. Given belief, $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$, this PBE where both the types will accept the proposed allocation and the government will repress when there is a protest will hold when, $w_0M > \tilde{\pi}_R^P > \hat{\pi}_R^P$.

Pooling equilibrium P.7:

Strong Protesters (R_s) and Weak Protesters: Reject & Protest

Government (G): Accommodate

G accommodates the protesters when there is protest when, $\frac{\alpha_0}{(1-\alpha_0)} > \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$.

Both the strong and the weak type of protesters (R_s, R_w) will reject the proposed allocation and protest when the payoff from protest under repression is more than the proposed allocation. Given belief, $\frac{\alpha_0}{(1-\alpha_0)} > \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$, this PBE where both the types of protesters will protest and the government will accommodate when there is a protest will hold when, $(w_2M - c_s^P) > (w_1M - c_w^P) > w_0M$.

Pooling equilibrium P.8:

Strong Protesters (R_s) and Weak Protesters: Accept

G accommodates the protesters when there is protest when, $\frac{\alpha_0}{(1-\alpha_0)} > \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$.

Both the strong and the weak type of protesters (R_s, R_w) will accept the proposed allocation when the payoff from protest under accommodation is less than the proposed allocation. Given belief, $\frac{\alpha_0}{(1-\alpha_0)} > \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$, this PBE where both the types of protesters will accept the proposed allocation and the government will accommodate the protesters when there is a protest will hold when, $w_0M > (w_2M - c_s^P) > (w_1M - c_w^P)$.

Separating equilibrium S.3:

Strong protesters (R_s): Reject & Protest.

Weak protesters (R_w): Accept.

Government(G): Beliefs in the protest information set $[R_s, R_w] = \{1, 0\}$, G believes that the strong protesters R_s protest. Given $h_s(v)^* < h(v) < h_w(v)^*$, G's best response is to accommodate the strong protesters (R_s). The strong protester will reject the proposed allocation and protest when, $w_2M - c_s^P > w_0M$. The weak protester will accept the proposed allocation and not deviate when $w_0M > w_1M - c_w^P$. Thus, this PBE will hold when the proposed allocation is such that, $(w_2M - c_s^P) > w_0M > (w_1M - c_w^P)$.

Similarly, the other case (of PBE) where the weak protesters R_w reject the proposed allocation and protest and the strong protesters (R_s) accept the proposed allocation and the government G believes that the weak protesters (R_w) will only protest, will not hold. If the weak protesters are better-off protesting under repression then the strong protesters must be better-off deviating and protesting under repression.

Let us consider the numerical example and see which equilibria holds;

Let $\gamma = \frac{1}{2}$, $w_0 = \frac{1}{4}, w_1 = \frac{1}{3}, w_2 = \frac{1}{2}$. Let $\frac{c_s}{M} = \frac{1}{10}$ and $\frac{c_w}{M} = \frac{2}{10}$. In this scenario for the no-pandemic case, equation (5.9) and (5.10) holds, thus the PBE where R_s protests, R_w accepts the proposed allocation and G believes that if there is a protest it is by the strong protesters (R_s) and accommodates the protesters will hold (Separating equilibrium A). In the post-pandemic scenario the virus thresholds (for both the types) are negative which implies for any level of virus spread G will be better-off accommodating the protesters. If the cost of protests in the presence pandemic rises extremely high for both the protesters then the PBE where the protesters will accept the proposed allocation will hold (Pooling equilibrium P.4). Whereas, if the cost of protest in the presence of pandemic does not increase substantially then the protesters will be better-off rejecting the proposed allocation and protesting (Pooling equilibrium P.3).

5.4.4 Equilibrium Refinements:

Let us see if Pooling equilibrium P.6 sustains under the Cho-Kreps (1987) intuitive criteria. Suppose the strong protesters(R_s) are better-off under accommodation $(w_2M - c_s^P) > w_0M$. The weak protesters are worse-off under repression and accommodation i.e., the equilibrium payoff is dominating for the weak protesters (R_w)[22], $w_0M > (w_1M - c_w^P) > \hat{\pi}_R^P$. If there is an off-the-equilibrium action of protest G will know it is the strong protesters and will accommodate them. The strong protesters (R_s) will be better-off under accommodation, so they will deviate and protest. This PBE will not sustain when the cost of protest for strong protesters is such that, $\left(\frac{k(v)M}{(1+k(v))^2} - w_0M\right) < c_s^P < (w_2 - w_0)M$ and the allocation after accommodation (for the strong protesters) is such that, $\frac{k(v)M}{(1+k(v))^2} < w_2 < \frac{(k(v))^2+2k(v)}{(1+k(v))^2}$. For the weak protesters the cost of protest is very high such that $c_w^P > (w_1 - w_0)M$ and the allocation after accommodation is such that $w_1 > \frac{(k(v)\gamma)^2+2\gamma k(v)}{(1+k(v)\gamma)^2} > \frac{(k(v)\gamma)^2}{(1+k(v)\gamma)^2}$.

Pooling equilibrium P.8 will always sustain under the Cho-Kreps (1987) intuitive criteria like in the no-pandemic case. Like the no-pandemic case, the reason follows that if the

[22] G is better-off repressing R_w when $w_1 > \frac{(k(v)\gamma)^2+2\gamma k(v)}{(1+k(v)\gamma)^2}$, which implies $w_1 > \frac{(k(v)\gamma)^2}{(1+k(v)\gamma)^2}$ and $\hat{\pi}_R^P = \frac{(k(v)\gamma)^2}{(1+k(v)\gamma)^2} - c_w^P$.

protesters are better-off in accepting the proposed allocation even when the government is willing to accommodate them then the equilibrium payoff is dominating for both the types.

Note: Considering the case, $h_w(v)^* < h_s(v)^*$ that is when $\gamma > \left(\frac{1-\sqrt{1-w_1}}{1-\sqrt{1-w_2}}\right)\left(\frac{\sqrt{1-w_2}}{\sqrt{1-w_1}}\right)$. When the virus spread is such that $h_w(v)^* < h(v) < h_s(v)^*$, the government is better-off repressing the strong type of protester and accommodating the weak type of protester. The PBE where both the types accept the proposed allocation and the government uses the repression strategy when there is a protest that is when, $w_0 M > \tilde{\pi}_R^P > \hat{\pi}_R^P$ and $\frac{\alpha_0}{(1-\alpha_0)} < \frac{\hat{\pi}_G^P-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G^P}$. This PBE will always sustain under the intuitive criteria, the reason being that the payoff for the strong protester is more than the weak protester both under repression and accommodation thus, it cannot be the case that the equilibrium payoff of acceptance is dominating for the strong protester but not so for the weak protester.

Observation 5.4. In the pandemic case compared to the no-pandemic case , the signaling equilibria depends on the level of the virus spread, the government (G) will repress both the types of protesters when the virus spread is very low ($h(v) < \min\{h_s(v)*, h_w(v)^*\}$) and will accommodate with both the types of protesters when the virus spread is very high ($h(v) > \max\{h(v_s^*), h(v_w^*)\}$). When the virus spread is between the two thresholds ($\min\{h(v_s^*), h(v_w^*)\} < h(v) < \max\{h(v_s^*), h(v_w^*)\}$), G will repress when the expected payoff from repression is more than that of accommodation otherwise will accommodate like the no-pandemic case. Like the no-pandemic case, the protesters will protest when the proposed allocation and cost of protest is low.

5.5 Protests in India

In this section we will discuss about the protests in India during the pandemic period and which of the signaling equilibria can best explain these protests.

5.5.1 Citizenship Amendment Act, India:

On 4th December 2019, after the approval of the CAA bill violent protests erupted in the north-eastern state of India, Assam. Soon after that protests were held in the metropolitan cities of India including Delhi, Mumbai, Bangalore and Kolkata. By the end of December 2019, the whole country was protesting against the act. The protest started before the pandemic announcement and continued after the World Health Organisation (WHO) declared the coronavirus outbreak as a global pandemic. On 24th March 2020, the Prime Minister of India called for a nation-wide lock-down to contain the spread of the coronavirus. The announcement of the lock-down along with strict containment measures marked the end of many protests including the month-long sit in protest in Shaheen Bagh in New-Delhi. So, we can say this protest was during the initial stages of the pandemic, so $h(v) < \min\{h(v_s^*), h(v_w^*)\}$ given any value of γ, the government's (G) best response is to repress the protesters. The repression of the anti-CAA protesters received worldwide condemnation. However, the protest was abandoned by the protesters and the government (G) has implemented the act. In this case we say that the proposed allocation is accepted by the protesters. We can say this is a pooling equilibrium[23] where the government's (G) best response is to repress and both the types of protesters (R_s, R_w) will accept the proposed the allocation instead of continuing to protest in response to repression, $w_0 M > \tilde{\pi}_R^P > \hat{\pi}_R^P$.

The anti-CAA protests was mainly carried out by students and working-class Muslim women. India also witnessed protests that was in favour of CAA and condemned the anti-CAA protests. In this case, we can categorise the protesters type as weak, R_w. We can say that this case also resonates with the separating equilibrium where the government (G) believes the if there is a continuing protest it is by the strong type (R_s) of protesters. Like said before the G's best-response is to repress. The strong type will continue with the protest in response to repression ($w_0 M < \tilde{\pi}_R^P$). The weak type (R_w) will be better-off accepting the proposed allocation ($w_0 M > \hat{\pi}_R^P$.).

5.5.2 Farmers' Protest, India:

The 2020–2021 Indian farmers' protest was against the three farm acts that were passed by the Parliament of India on September 2020. These acts were believed to leave the farmers at the "mercy of corporate". The protests also demanded the creation of a minimum support price (MSP)[24]bill, to ensure that corporates cannot control the prices[25].When the government first introduced this reform policy in June 2020, demonstrations initially sparked in the northern states of India. Samyukt Kisan Morcha (SKM)

[23]When we say equilibrium we are basically trying to emphasize the fact whether the protesters are continuing with the protest or not even after repression or accommodation.

[24]Minimum support price (MSP) is a market intervention by the Government of India to insure farmers against price volatility. It is announced by the Government of India at the beginning of the sowing season for specific crops as per the the recommendations of the Commission for Agricultural Costs and Prices (CACP).

[25]https://frontline.thehindu.com/the-nation/agriculture/india-at-75-epochal-moments-2020-farmers-protests-take-the-country-by-storm/article65722271.ece

along with the Bharatiya Kisan Union (BKU) acted as a key umbrella organization for about 40 national and regional farmer unions. In November 2020, thousands of protesters set up camps on highways leading to New Delhi when the SKM organized nationwide protests. During this time India was battling the second and most brutal wave of coronavirus infections. It broke the record for maximum number of daily infections and deaths[26]. So we can say that this protest was during the peak stage of the pandemic thus, $h(v) > \max\{h(v_s^*, h(v_w^*))\}$ given any value of γ, G's best response is to accommodate the farmers. The government (G) tried to accommodate the farmers by suspending the acts for 18 months which was rejected by the farmers[27]. The farmers continued with the protest till the government repealed the three acts. We can say this is the pooling equilibrium where the government's (G) best response is to accommodate the protesters and both the types of protesters (R_s, R_w) will protest, $(w_2M - c_s^P) > (w_1M - c_w^P) > w_0M$.

Despite several clashes between police and protesters, ACLED report shows that since the onset of the farmers movement in June 2020, almost 98% of the demonstrations related with the farmers movement have been non-violent[28]. Since the passing of the three farm acts, the farmer movement sustained its strength and appeal for over a year. Hence, we can categorize protesters as strong (R_s). We can say that this is the separating equilibrium where the government (G) believes that if there is a protest it is by the strong protesters hence its best response is to accommodate ($h(v) > \max\{h(v_s^*, h(v_w^*))\}$). The strong type protesters (R_s) will reject the government's (G) proposed allocation and continue with their protest whereas the weak type will accept the proposed allocation, $w_1M - c_w^P < w_0M < w_2M - c_s^P$.

5.6 Conclusion

One of the essential components of a democracy is the right to freedom of peaceful assembly and association which includes right to organize meetings, rallies, strikes or protest. History is a witness of how protests have acted as a driving force behind some of most significant social movements. From women suffrage movement to 1930's Salt March to protest against the colonial salt tax in India, protests have had significant impacts on policy making to overthrowing the government in power. From a rational perspective, protest and repression has been interrelated strategic decisions of the players.

[26]https://www.aljazeera.com/news/2021/4/26/indian-farmers-continue-anti-farm-law-protests-amid-covid-surge

[27]This can shown as an additional level in the game where government's (G) second proposal can be rejected or accepted by the protesters.

[28]https://acleddata.com/2021/12/17/an-unlikely-success-demonstrations-against-the-farm-laws-in-india/

The COVID-19 pandemic had an unforgettable impact on society. During the pandemic the world witnessed an increase in violence and protests. India witnessed a number of protest mobilisations during the pandemic years. With the farmers protest in the backdrop, there were protest and strikes carried out by the working class of India. The Accredited Social Health Activists(ASHA) and Anganwadi workers called out nationwide strikes in August 2020, May 2021 and September 2021 demanding wages, social security and acknowledgement especially focussing on their efforts during pandemic[29]. During this same period, transport workers, bank employees, and frontline medical workers also conducted strikes, rallies, and protests either individually or in support of nationwide movements (Surendran et al., 2024). This chapter does not discuss about these protests because these protests are not against the government's proposed policy or allocation.

This chapter is written keeping in mind the two significant protests India witnessed during the pandemic period. It is an attempt to study how the pandemic can affect the the strategic behaviour of government and protesters during a protest. It shows that at the initial stages of the pandemic the government might prefer to repress the protesters and since the pandemic also increases the cost to protest for the protesters, the protesters may prefer not to continue with the protest. However on the other hand, repression of protests becomes costly for the government since the pandemic imposes an additional cost on the government. Hence, with the increase in virus spread the fighting efforts of both the government and the protesters starts falling and when the virus spread increases beyond a certain threshold the government is better-off accommodating with the protesters. Comparing the pandemic signaling equilibria with the benchmark case of no-pandemic signaling equilibria, we observed that the players decision to protest and repress depends on the level of virus spread.

[29]https://www.hindustantimes.com/india-news/asha-anganwadi-workers-to-go-on-nationwide-strike-today-101632451397778.html

Chapter 6

Conclusion

This dissertation expands the existing literature on conflict dynamics by examining how revenge motives and third-party intervention influence the strategic behaviour of combatants. Additionally, it explores the interaction between protesters and government during a protest, specifically looking into how their behaviour changes in response to an external shock like the COVID-19 pandemic.

Chapter 3 analyses how the presence of a third-party in a conflict can influence the level of conflict with or without revenge. It shows that the combatants are better off exacting revenge on each other, irrespective of whether the third party has intervened or not (either as an ally of one of them or as an 'idealist'). The third party intervenes as an ally only when the (strategic) value of the resource, if its ally wins, is sufficiently high. While the third party intervenes as an idealist only if its intervention can significantly mitigate resultant conflict levels.

Chapter 3 is just intended to be a preliminary investigation into the issue of third party intervention and revenge. There are plenty of avenues to explore in this area. For example, Munshi (2021) explored the possible impact of the pandemic on conflicts all over the world. She found that under most parametric restrictions conflicts are likely to fall due to the ongoing pandemic. How will her results be affected when there are revenge motivations and/or third party interventions? For example, the way conflicts are affected during a pandemic in Munshi (2021), is through spillovers or externalities between countries, both positive and negative (for example, vaccines are exported and imported which may constitute a positive externality whereas people travelling between countries, who are thereby acting as potential carriers of the virus, may constitute a negative externality). In the presence of revenge motivations and/or third party interventions, the presence and effectiveness of these externalities will be affected. Hence, close examination of how these channels are impacted will be necessary to see how conflicts will be affected in general.

Chapter 4 provides a new approach of desire-capability revenge model of conflict. It shows that a combatant will engage in a second conflict (out of revenge) only when it has the capability to so given the destruction suffered in the first period conflict. This chapter opens up many more avenues to explore. Like, Liang et al. (2020) re-examined the paradox of revenge in a defender-attacker scenario and as per their model only the defender takes revenge in the following period. How will their results change in this desire-capability function of revenge? What happens when the defender's type is imperfectly observed by the attacker i.e., the attacker is not completely informed about the capability of the defender.

One of the future extension of this model is to study how a third-party can influence the strategies of the combatants in this revenge-capability model of conflict. At the end of chapter 4, I have briefly discussed that whether a third-party can help its ally in the conflict such that its opponent is completely incapacitated. I have also briefly discussed how the pandemic impacts the strategies of the combatants in this framework.

Chapter 6 is an attempt to study how the players (government and protesters) will interact in a protest during a pandemic using a signaling game where the protesters' type is imperfectly observed by the government. This chapter compares the the benchmark case of no-pandemic with the pandemic case. This model shows how the virus spread influences the strategies of the players and the intensity of the protests.

Chapter 6 does not attempt to model a convincing collective action during a pandemic. There are literature on social networks (Gould, 1993; Siegel, 2009) highlighting the usefulness of network structure, behavioural standards, and the position of activists for successful collective action. However, this chapter is an attempt to model protests during the pandemic using a simple sequential game of incomplete information, specifically keeping the recent protests in India in mind.

Appendix A

Appendix: Chapter 3

A.0.1 With revenge, but no third-party case:

The equilibrium levels of period 3 conflict investments, $\hat{x}_{A3}$ and $\hat{x}_{B3}$ (Amegashie and Runkel (2012) call this the 'value-effect' of revenge) are as follows:

$$\hat{x}_{i3} = \frac{R_i^2 R_j}{(R_A + R_B)^2}, \quad i,j = A,B, \quad i \neq j, \tag{A.1}$$

The third period equilibrium utility as a function of second period conflict investment levels can be calculated as follows:

$$\hat{\pi}_{i3} = \frac{R_i^3}{(R_A + R_B)^2}, \quad i = A,B; \tag{A.2}$$

To know the impact of player i's second period effort on player i's third period equilibrium payoff, we differentiate $\hat{\pi}_{i3}$ w.r.t x_{i2}, as follows:

$$\frac{d\hat{\pi}_{i3}}{dx_{i2}} = -\frac{2R_i^3 R_j'}{(R_A + R_B)^3} < 0, \quad i \neq j, \quad i = A,B; \tag{A.3}$$

Hence an increase in player i's effort in the second period reduces player i's payoff of period 3, because increase in i's second period efforts also increases player j's revenge efforts in period 3, thereby reducing i's period 3 payoff. The payoff function in period 2 is the present value (without discounting) of the payoffs from both the periods, as given below:

$$\pi_{i2} = \frac{x_{i2}}{x_{i2} + x_{j2}} V_i - x_{i2} + \hat{\pi}_{i3}, i = A,B; i \neq j. \tag{A.4}$$

Maximising w.r.t x_{i2}, we get the equilibrium levels of period 2 conflict investments, $\hat{x}_{A2}$

and $\hat{x}_{B2}$. Comparing the FOCs after differentiating payoff functions without revenge ($\pi_{i2} = \frac{x_{i2}}{x_{i2}+x_{j2}}V_i - x_{i2}$) and with revenge (equation (A.4)) w.r.t x_{i2}, we see that the conflict effort value for which (A.4) is maximised is lower than the conflict effort value for which $\pi_{i2}(= \frac{x_{i2}}{x_{i2}+x_{j2}}V_i - x_{i2})$ is maximised. Amegashie and Runkel (2012) call this the 'self-deterrence effect' of revenge.

A.0.2 Proposition 3.1 has the condition $(1+M)^\theta < 2$. To exhaustively cover the possibilities we lay down Proposition 3.7 that has condition $(1+M)^\theta > 2$.

Proposition 3.7: *Let* $R_i(x_{j2}) = \alpha x_{j2}^\phi, i \neq j, V_A = V_B$ *and* $(1+M)^\theta > 2$. *Then we have the following:*

$$(i)\ \ X_R^T > X_T > \tilde{X},\ \ \textit{and}\ \ X_R^T > \hat{X} > \tilde{X}\ \ \textit{when}\ \ \phi < \frac{1}{(1+M)^\theta},$$
$$(ii)\ \ X_T > X_R^T > \hat{X} > \tilde{X},\ \ \textit{when}\ \ 1 > \phi > \frac{1}{2},$$
$$(iii)\ \ X_T > X_R^T > \hat{X},\ \ \textit{and}\ \ X_I > \tilde{X} > \hat{X},\ \ \textit{when}\ \ \phi > 1.$$

The proof below is the proof of both the propositions.

Proof. We proceed by backward induction. The third period equilibrium values, given in equation (A.1) and from equation (A.3) we know the impact of second period effort on the third period equilibrium payoff of the respective combatants.

The aggregate third period conflict is;

$$\hat{X}_3 = \frac{\alpha(x_{A2}x_{B2})^\phi}{(x_{A2}^\phi + x_{B2}^\phi)}$$

$$\frac{d\hat{\pi}_{A3}}{dx_{A2}} = -\frac{2R_A^3 R_B'}{(R_A+R_B)^3} = -\frac{2\alpha(x_{B2}^{3\phi})(\phi x_{A2}^{\phi-1})}{(x_{B2}^\phi + x_{A2}^\phi)^3}$$

Similarly, for combatant B,

Now, coming to the second period FOCs, for A and B respectively, and writing it for this revenge framework, we get;

$$\frac{d\pi_{A2}}{dx_{A2}} = \frac{x_{B2}V}{(x_{A2}+x_{B2})^2} - \frac{1}{(1+M)^\theta} - \frac{2\alpha(x_{B2}^{3\phi})(\phi x_{A2}^{\phi-1})}{(x_{B2}^\phi + x_{A2}^\phi)^3} = 0 \tag{A.5}$$

$$\frac{d\pi_{B2}}{dx_{B2}} = \frac{x_{A2}V}{(x_{A2}+x_{B2})^2} - 1 - \frac{2\alpha(x_{A2}^{3\phi})(\phi x_{B2}^{\phi-1})}{(x_{B2}^{\phi}+x_{A2}^{\phi})^3} = 0 \tag{A.6}$$

Now, let us write equation (A.5) in the following way:

$$\frac{1}{(1+M)^{\theta}} = \frac{x_{B2}V}{(x_{A2}+x_{B2})^2} - \frac{2\alpha(x_{B2}^{3\phi})(\phi x_{A2}^{\phi-1})}{(x_{B2}^{\phi}+x_{A2}^{\phi})^3}$$

Now, multiplying both sides with x_{A2}.

$$\frac{x_{A2}}{(1+M)^{\theta}} = \frac{x_{B2}x_{A2}V}{(x_{A2}+x_{B2})^2} - \frac{2\alpha(x_{B2}^{3\phi})(\phi x_{A2}^{\phi})}{(x_{B2}^{\phi}+x_{A2}^{\phi})^3} \tag{A.7}$$

Similarly, equation (A.6) can be written, by multiplying both sides with x_{B2}.

$$x_{B2} = \frac{x_{B2}x_{A2}V}{(x_{A2}+x_{B2})^2} - \frac{2\alpha(x_{A2}^{3\phi})(\phi x_{B2}^{\phi})}{(x_{B2}^{\phi}+x_{A2}^{\phi})^3} \tag{A.8}$$

Subtracting, equation (A.8) from (A.7), and simplifying we get

$$\frac{x_{A2}}{(1+M)^{\theta}} - x_{B2} + \frac{2\alpha\phi x_{A2}^{\phi}x_{B2}^{\phi}(x_{A2}^{2\phi}-x_{B2}^{2\phi})}{(x_{B2}^{\phi}+x_{A2}^{\phi})^3} = 0 \tag{A.9}$$

Now, when we put $x_{A2} = (1+M)^{\theta}x_{B2}$, equation (**??**) becomes;

$$0 + \frac{2\alpha\phi x_{A2}^{\phi}(1-\frac{1}{(1+M)^{2\theta\phi}})}{(1+M)^{\theta\phi}(1+\frac{1}{(1+M)^{\theta\phi}})^3} = 0 \tag{A.10}$$

The power of $(1+M)$ rises and the max value of $\frac{1}{(1+M)^{2\theta\phi}}$ is 1. And the value of the denominator also rises, therefore we can approximate the value of the second term in equation (A.9) to 0. In short, this is a possible approximation[1] of equation (A.10). Thus, $x_{A2}^{TR} = (1+M)^{\theta}x_{B2}^{TR}$.

Now, period 2 conflict is $X_2^{TR} = x_{A2}^{TR} + x_{B2}^{TR}$. And by putting the value of x_{B2}^{TR} in terms of x_{A2}^{TR}, we get;

$$X_2^{TR} = \frac{V(1+M)^{\theta}}{1+(1+M)^{\theta}} - (\frac{2\alpha\phi x_{A2}^{\phi}}{((1+M)^{\theta\phi})(1+\frac{1}{(1+M)^{\theta\phi}})})(1+\frac{(1+M)^{\theta}}{(1+M)^{2\theta\phi}}) \tag{A.11}$$

[1]This is not a complete solution. Since this chapter assumes third-party intervention, there are many parameters involved so we have worked out the most closest solution using a possible approximation.

The third period conflict written in terms of x_{A2}^{TR},

$$\hat{X}_3 = \frac{\alpha x_{A2}^{\phi}}{(1+M)^{\theta\phi}(1+\frac{1}{(1+M)^{\theta\phi}})} \tag{A.12}$$

Aggregate level of conflict $X_R^T = X_2^{TR} + \hat{X}_3$.

Now, subtracting X_T ($X_T = \frac{V_A(1+M)^{\theta}}{((1+M)^{\theta}+1)}$) from X_R^T.

$$X_R^T - X_T = \frac{\alpha x_{A2}^{\phi}}{(1+M)^{\theta\phi}(1+\frac{1}{(1+M)^{\theta\phi}})}(1-2\phi(\frac{1+(1+M)^{\theta(1-2\phi)}}{(1+\frac{1}{(1+M)^{\theta\phi}})^2})) \tag{A.13}$$

In order to compare the two conflict levels we have to check the equation in the parenthesis.

$$1 > 2\phi(\frac{(1+M)^{2\theta\phi}+(1+M)^{\theta}}{(1+(1+M)^{\theta\phi})^2}) \tag{A.14}$$

Adding an extra restriction of $(1+M)^{\theta} < 2$ (similarly, adding restriction $(1+M)^{\theta} > 2$ in Proposition 3.7), because though the revenge function is symmetric in this proposition, for the third party case it is not because the maximizing equations involve a third party acting as an ally of A. When $\phi < \frac{1}{2}$ equation (A.14) is positive. No paradox of revenge for low values of ϕ. Now, when $\phi > \frac{1}{(1+M)^{\theta}}$, equation (A.13) is negative. The paradox of revenge holds. ■

A.0.3 Proof of Proposition 3.2

Proof. Proceeding like Proposition 3.1; The third period conflict level is;

$$\hat{X}_3 = \frac{\alpha_A \alpha_B x_{A2} x_{B2}}{\alpha_A x_{B2} + \alpha_B x_{A2}}$$

Like Proposition 3.1, the second period can be written as;

$$\frac{x_{A2}}{(1+M)^{\theta}} = \frac{x_{B2}x_{A2}V}{(x_{A2}+x_{B2})^2} - \frac{2\alpha_B x_{A2}(\alpha_A x_{B2})^3}{(\alpha_A x_{B2} + \alpha_B x_{A2})^3}$$

$$x_{B2} = \frac{x_{B2}x_{A2}V}{(x_{A2}+x_{B2})^2} - \frac{2\alpha_A x_{B2}(\alpha_B x_{A2})^3}{(\alpha_A x_{B2} + \alpha_B x_{A2})^3}$$

Total conflict $X_R^T = X_2^{TR} + \hat{X}_3$.

$$X_R^T = \frac{(1+(1+M)^\theta)x_{A2}x_{B2}}{(x_{A2}+x_{B2})^2} + \frac{\alpha_A\alpha_B x_{A2}x_{B2}((\alpha_A x_{B2}+\alpha_B x_{A2})^2 - 2(1+M)^\theta(\alpha_A x_{B2})^2 - 2(\alpha_B x_{A2})^2)}{(\alpha_A x_{B2}+\alpha_B x_{A2})^3}$$

Now, subtracting X_T ($X_T = \frac{V(1+M)^\theta}{((1+M)^\theta+1)}$) from X_R^T.

$$X_R^T - X_T = \frac{(1+(1+M)^\theta)x_{A2}x_{B2}}{(x_{A2}+x_{B2})^2} + \frac{\alpha_A\alpha_B x_{A2}x_{B2}((\alpha_A x_{B2}+\alpha_B x_{A2})^2 - 2(1+M)^\theta(\alpha_A x_{B2})^2 - 2(\alpha_B x_{A2})^2)}{(\alpha_A x_{B2}+\alpha_B x_{A2})^3} - \frac{V(1+M)^\theta}{1+(1+M)^\theta} \tag{A.15}$$

Writing equation (A.15) in the following way;

$$X_R^T - X_T = \frac{V((1+(1+M)^\theta)^2 x_{A2}x_{B2} - (1+M)^\theta(x_{A2}+x_{B2})^2)}{(1+(1+M)^\theta)(x_{A2}+x_{B2})^2} + \frac{\alpha_A\alpha_B x_{A2}x_{B2}((\alpha_A x_{B2}+\alpha_B x_{A2})^2 - 2(1+M)^\theta(\alpha_A x_{B2})^2 - 2(\alpha_B x_{A2})^2)}{(\alpha_A x_{B2}+\alpha_B x_{A2})^3} \tag{A.16}$$

Let us check the first part of RHS of equation (A.16)

Further simplifying and considering $(1+M)^{2\theta} \approx (1+M)^\theta$ and given $(1+M)^\theta < 1$, we get;

$$\frac{-V(1+((1+M)^\theta)((x_{A2})^2+(x_{B2})^2 - x_{A2}x_{B2}(1+\frac{1}{(1+M)^\theta}))}{(1+(1+M)^\theta)(x_{A2}+x_{B2})^2} < 0 \tag{A.17}$$

The result comes from $((x_{A2})^2+(x_{B2})^2 - x_{A2}x_{B2}(1+\frac{1}{(1+M)^\theta})) > ((x_{A2})^2+(x_{B2})^2 - 2x_{A2}x_{B2}) > 0$.

Simplifying the second part of RHS of equation (A.16);

$$\frac{\alpha_A\alpha_B x_{A2}x_{B2}((1-2(1+M)^\theta)(\alpha_A x_{B2})^2 - (\alpha_B x_{A2})^2 + 2\alpha_A\alpha_B x_{A2}x_{B2})}{(\alpha_A x_{B2}+\alpha_B x_{A2})^3} \tag{A.18}$$

The parenthesis part in the numerator of equation (A.18) is what we to need to check.

$$-((2(1+M)^\theta - 1)(\alpha_A x_{B2})^2 + (\alpha_B x_{A2})^2 - 2\alpha_A\alpha_B x_{A2}x_{B2}) < 0$$

Now, $(2(1+M)^\theta - 1) > 1$, thus we get;

$((2(1+M)^\theta - 1)(\alpha_A x_{B2})^2 + (\alpha_B x_{A2})^2 - 2\alpha_A\alpha_B x_{A2}x_{B2}) > ((\alpha_A x_{B2})^2 + (\alpha_B x_{A2})^2 - 2\alpha_A\alpha_B x_{A2}x_{B2}) > 0$.

Therefore equation (A.16) is negative and we get $X_R^T < X_T$, the paradox of revenge holds. ∎

A.0.4 Proof of Proposition 3.3

Proof. The third period conflict level is;

$$\hat{X}_3 = \frac{x_{A2}x_{B2}}{x_{B2} + x_{A2}}$$

The second period FOCs are;

$$x_{A2} = \frac{x_{A2}x_{B2}(1+M)^\theta(V_A(x_{A2}+x_{B2}) - 2x_{B2}^2)}{(x_{A2}+x_{B2})^3} \tag{A.19}$$

$$x_{B2} = \frac{x_{A2}x_{B2}(V_B(x_{A2}+x_{B2}) - 2x_{A2}^2)}{(x_{A2}+x_{B2})^3} \tag{A.20}$$

Total conflict $X_R^T = x_{A2} + x_{B2} + \hat{X}_3$. Total conflict with third party intervention without revenge $X_T = \frac{V_A V_B (1+M)^\theta}{V_B + V_A(1+M)^\theta}$. Checking $X_R^T - X_T$.

$$X_R^T - X_T = \left(\frac{x_{A2}x_{B2}(V_B + V_A(1+M)^\theta)}{(x_{A2}+x_{B2})^2} - \frac{V_A V_B (1+M)^\theta}{V_B + V_A(1+M)^\theta}\right) + \left(\frac{x_{A2}x_{B2}((x_{A2}+x_{B2})^2 - 2(x_{A2}^2 + x_{B2}^2(1+M)^\theta))}{(x_{A2}+x_{B2})^3}\right) \tag{A.21}$$

We will check both the terms in the RHS of equation (A.21). The second term of the RHS is negative (solving like Proposition 3.2).

The first term can be written in the following way;

$$\frac{x_{A2}x_{B2}(V_B + V_A(1+M)^\theta)^2 - V_A V_B (1+M)^\theta (x_{A2}+x_{B2})^2}{(x_{A2}+x_{B2})^2(V_B + V_A(1+M)^\theta)} \tag{A.22}$$

The numerator of equation (A.22) can be simplified and written;

$$(x_{B2}V_B - x_{A2}V_A)(x_{A2}V_B - x_{B2}V_A(1+M)^\theta) \tag{A.23}$$

Now, subtracting equation (A.20) from (A.19);

$$x_{A2} - x_{B2} \approx \frac{x_{A2}x_{B2}(V_A(1+M)^\theta - V_B)}{x_{A2}^2 + x_{B2}^2}. \tag{A.24}$$

Without loss of generality let us assume $V_A > V_B \implies V_A(1+M)^\theta > V_B$. From equation (A.24) then we get $x_{A2} > x_{B2}$. Thus, we get

$$x_{A2}V_A > x_{B2}V_B \tag{A.25}$$

Let us multiply V_B on both sides of equation (A.19) and $V_A(1+M)^\theta$ on both sides of equation (A.20), then subtracting it we get;

$$x_{A2}V_B - x_{B2}V_A(1+M)^\theta = 2(x_{A2}^2 V_A - x_{B2}^2 V_B) \tag{A.26}$$

Given the condition $V_A > V_B$, it implies $x_{A2}^2 V_A > x_{B2}^2 V_B$, therefore,

$$x_{A2}V_B > x_{B2}V_A(1+M)^\theta \tag{A.27}$$

From equations (A.25) and (A.27) we get two opposite signs for the two parts of equation (A.23) when $V_A > V_B$. Similarly when $V_B > V_A$.

Therefore, the two parts of equation (A.21) are negative. Thus $X_R^T < X_T$, the paradox of revenge holds. ∎

A.0.5 Proof of Proposition 3.4

Proof. In the idealist part, we have the same third period conflict only the second period FOCs changes.

The second period FOCs are;

$$P(M) = \frac{x_{B2}V}{(x_{A2}+x_{B2})^2} - \frac{2R_A^3\phi\alpha x_{A2}^{\phi-1}}{(R_A+R_B)^3} \tag{A.28}$$

$$P(M) = \frac{x_{A2}V}{(x_{A2} + x_{B2})^2} - \frac{2R_B^3 \phi \alpha x_{B2}^{\phi-1}}{(R_A + R_B)^3} \tag{A.29}$$

Multiplying equation (A.28) with x_{A2} and equation (A.29) with x_{B1} and then subtracting it we get;

$$P(M)(x_{A2} - x_{B2}) = \frac{2\phi\alpha^4 (x_{A2}x_{B2})^{\phi}(x_{A2}^{2\phi} - x_{B2}^{2\phi})}{(R_A + R_B)^3}$$

Thus, $x_{A2} = x_{B2} = x_2$.

Total conflict $X_R^I = 2x_2 + \hat{X}_3$ and $\hat{X}_3 = \frac{\alpha x_2^{\phi}}{2}$.

$$X_R^I - X_I = \frac{\alpha x_2^{\phi}(1 - \frac{\phi}{P(M)})}{2}$$

Now, $X_R^I > X_I$ when $\phi < P(M)$, no paradox of revenge. $X_I > X_R^I$ when $\phi > P(M)$, paradox of revenge holds. ■

Appendix B

Appendix: Chapter 4

B.0.1 When desire for revenge is sensitive to the destruction suffered in the previous period

In this framework the revenge-capability function becomes, $R_i = x_{j1}^{\phi} * (\alpha_i - \gamma_i x_{j1})$.

The revenge-capability function can be written in the following way (considering the revenge-capability function of combatant A);

$$R_A = x_{B1}^{\phi}(\alpha_A - \gamma_A x_{B1}) > 0 \text{ when } 0 < x_{B1} < \frac{\phi \alpha_A}{(1+\phi)\gamma_A}. \tag{B.1}$$

$$R_A = 0 \text{ when } x_{B1} = 0. \tag{B.2}$$

$$R_A = 0 \text{ when } x_{B1} \geq \frac{\alpha_A}{\gamma_A}. \tag{B.3}$$

Like before when $x_{B1} \geq \frac{\alpha_A}{\gamma_A}$ combatant B has totally incapacitated its opponent A in the first period conflict and there won't be any second period conflict out of revenge.

The R_P^A point is $\frac{\phi \alpha_A}{(1+\phi)\gamma_A}$.

If the desire for revenge is more sensitive to the destruction suffered in the previous period that is $\phi > 1$ then the R_P^A point of A will be high. Thus the level of incapacitation where it would have less incentive to retaliate is more for the combatant. A is very revengeful and its desire for revenge will fall only when the incapacitation will be at a high level.

The less the sensitivity ($\phi < 1$), the lower the R_P point and thus the incapacitation to level where it would have less incentive to retaliate is less.

If the sensitivity to previous destruction is low such that both the combatants can reach its opponent's revenge peak point point then from Proposition 4.5 we know that the intensity of the conflict falls with time. Thus, when the elasticity is low, the urge to go into second period conflict out of revenge depends mostly on the capability factor and otherwise when the sensitivity is high.

B.0.2 Proof of Proposition 4.1

$$X_R - \tilde{X} = \frac{2x_{B1}x_{A1}V}{(x_{A1}+x_{B1})^2} - (\frac{2R_B^3 x_{B1}(\alpha_A - 2\gamma_A x_{B1})}{(R_A+R_B)^3} + \frac{2R_A^3 x_{A1}(\alpha_B - 2\gamma_B x_{A1})}{(R_A+R_B)^3}) + \frac{R_A R_B}{(R_A+R_B)} - \frac{V}{2}. \quad \text{(B.4)}$$

$$X_R - \tilde{X} = (\frac{2x_{B1}x_{A1}V}{(x_{A1}+x_{B1})^2} - \frac{V}{2}) + (\frac{R_A R_B}{(R_A+R_B)} - (\frac{2R_B^3 x_{B1}(\alpha_A - 2\gamma_A x_{B1})}{(R_A+R_B)^3} + \frac{2R_A^3 x_{A1}(\alpha_B - 2\gamma_B x_{A1})}{(R_A+R_B)^3})). \quad \text{(B.5)}$$

$$X_R - \tilde{X} = \frac{-V(x_{A1}-x_{B1})^2}{2(x_{A1}+x_{B1})^2} + (\frac{R_A R_B}{(R_A+R_B)} - (\frac{2R_B^3 x_{B1}(\alpha_A - 2\gamma_A x_{B1})}{(R_A+R_B)^3} + \frac{2R_A^3 x_{A1}(\alpha_B - 2\gamma_B x_{A1})}{(R_A+R_B)^3})). \quad \text{(B.6)}$$

We have to check for the second term in equation (B.6), since the first term is negative, as shown.

The second part of the equation can be written as:

$$\frac{R_A R_B (R_A+R_B)^2 - 2(R_A^3 x_{A1}(\alpha_B - 2\gamma_B x_{A1}) + R_B^3 x_{B1}(\alpha_A - 2\gamma_A x_{B1}))}{(R_A+R_B)^3}. \quad \text{(B.7)}$$

$$\frac{R_A R_B (R_A+R_B)^2 - 2(R_A^3(R_B - \gamma_B x_{A1}^2) + R_B^3(R_A - \gamma_A x_{B1}^2))}{(R_A+R_B)^3}. \quad \text{(B.8)}$$

Further simplifying the equations;

$$\frac{2(R_A R_B)^2 - R_A^3 x_{A1}(\alpha_B - 3\gamma_B x_{A1}) - R_B^3 x_{B1}(\alpha_A - 3\gamma_A x_{B1})}{(R_A+R_B)^3}. \quad \text{(B.9)}$$

$$\frac{(R_A^2(R_B^2 - R_A x_{A1}(\alpha_B - 3\gamma_B x_{A1})) + (R_B^2(R_A^2 - R_B x_{B1}(\alpha_A - 3\gamma_A x_{B1})))}{(R_A + R_B)^3}. \quad \text{(B.10)}$$

Let $x_{A1} < \frac{\alpha_B}{3\gamma_B}, x_{B1} < \frac{\alpha_A}{3\gamma_B}$, and $R_A > R_B$ such that $R_B^2 < R_A x_{A1}(\alpha_B - 3\gamma_B x_{A1})$, which makes the first term negative. The second term of equation (B.10) is positive thus if R_A is sufficiently large that it dominates the positive value of the second term, making the whole equation (B.10) negative. Thus, the paradox of revenge can be observed. Similarly, when $R_B > R_A$.

B.0.3 Proof of Proposition 4.2

First period FOCs of the combatants can be written in the following way:

$$x_{i1} = \frac{x_{B1}x_{A1}V}{(x_{A1} + x_{B1})^2} - \frac{2R_i^3 x_{i1}(\alpha_j - 2\gamma_j x_{i1})}{(R_A + R_B)^3}, \text{ where } i \neq j \text{ and } i = A, B. \quad \text{(B.11)}$$

We can write (B.11) in the following way:

$$x_{A1} = x_{B1} + \frac{2R_B^3 x_{B1}(\alpha - 2\gamma x_{B1}) - 2R_A^3 x_{A1}(\alpha - 2\gamma x_{A1})}{(R_A + R_B)^3}. \quad \text{(B.12)}$$

When $x_{A1} = x_{B1}$ LHS=RHS of equation (B.12).

$$\hat{x}_{A1} = \hat{x}_{B1} = \hat{x}_1 = \frac{V}{4} - \frac{\hat{x}_1(\alpha - 2\gamma\hat{x}_1)}{4}. \quad \text{(B.13)}$$

Total conflict X_R is;

$$X_R = 2x + \frac{R_A R_B}{(R_A + R_B)} = \frac{V}{2} - \frac{\hat{x}_1(\alpha - 2\gamma\hat{x}_1)}{2} + \frac{\hat{x}_1(\alpha - \gamma\hat{x}_1)}{2}. \quad \text{(B.14)}$$

$$X_R = \frac{V}{2} + \frac{\hat{x}_1^2\gamma}{2}. \quad \text{(B.15)}$$

B.0.4 Proof of proposition 4.3

In the benchmark case of no revenge, the payoffs are as follows:

$$\tilde{\pi}_{A1} = \tilde{\pi}_{B1} = \frac{V}{4}. \quad \text{(B.16)}$$

The payoffs of A and B in the presence of revenge are as follows:

$$\hat{\pi}_{A1} = \frac{x_{A1}}{(x_{A1} + x_{B1})}V - x_{A1} + \hat{\pi}_{A2}, \quad \text{(B.17)}$$

$$\hat{\pi}_{B1} = \frac{x_{B1}}{(x_{A1} + x_{B1})}V - x_{B1} + \hat{\pi}_{B2}. \quad \text{(B.18)}$$

Comparing these with the payoffs in the benchmark case of no revenge, we get:

$$\hat{\pi}_{A1} - \tilde{\pi}_{A1} = -\frac{V(\hat{x}_{B1} - 3\hat{x}_{A1})}{4(\hat{x}_{A1} + \hat{x}_{B1})} - \hat{x}_{A1} + \frac{R_A^3}{(R_A + R_B)^2}. \quad \text{(B.19)}$$

From equation (B.19) it is evident that if the first period equilibrium conflict investment of A is relatively very high from the first period conflict investment of B then the payoff of A is more in the presence of revenge. However, if the equilibrium conflict investment of B is relatively very high than that of A then payoff of A will be lower in the presence of revenge than without revenge. This shows if the B has incapacitated A to a high level in the first period conflict then A will not want to go into a second period conflict out of revenge. The comparison is similar for that of B.

When the capabilities of the combatants is similar we already know from Proposition 4.2 that the first period conflict investments of the combatants are equal and thus equation (B.19) becomes;

$$\hat{\pi}_{A1} - \tilde{\pi}_{A1} = (\frac{V}{4} - \hat{x}_1) + \frac{R_A^3}{(R_A + R_B)^2}. \quad \text{(B.20)}$$

From Proposition 4.2 we know the value of $\hat{x}_1$ and from there we can write;

$$\frac{V}{4} - \hat{x}_1 = \frac{\hat{x}_1(\alpha - 2\gamma\hat{x}_1)}{4}. \quad \text{(B.21)}$$

Now, putting this value of $(\frac{V}{4} - \hat{x}_1)$ in (B.20) we get:

$$\hat{\pi}_{A1} - \tilde{\pi}_{A1} = \frac{\hat{x}_1(\alpha - 2\gamma\hat{x}_1)}{4} + \frac{R_A^3}{(R_A + R_B)^2}. \quad \text{(B.22)}$$

When $\hat{x}_1 < \frac{\alpha}{2\gamma}$ then equation (B.21) becomes positive which makes equation (B.20) positive. Which shows that both the combatants are better off in the presence of revenge when both the combatants are not able to incapacitate each other significantly in the first period conflict. Thus, when the revenge function of both the combatants are rising both of the combatants are willing to go into a second period conflict out of revenge.

If $\hat{x}_1 > \frac{\alpha}{2\gamma}$ then equation (B.21) becomes negative and if this negative value is high enough such that it negates the positive desire of exacting revenge, then equation (B.20) becomes negative. Thus, both the combatants are worse off in the presence of revenge when both the combatants are capable enough to incapacitate each other to a high level in the first period conflict.

Now, let us calculate the value of the first period conflict investment in the symmetric case when the combatants would be (better-off) worse-off in the presence of revenge.

$$\hat{\pi}_{A2} = \frac{R_A^3}{(R_A + R_B)^2} = \frac{R}{4} = \frac{\hat{x}_1(\alpha - \gamma\hat{x}_1)}{4}. \tag{B.23}$$

Since this is a symmetric case and the equilibrium payoff of A equals that of B so re-writing (B.22) in terms of i where $i = A, B$ and putting the value of $\hat{\pi}_{A2}$ in the equation, we get the following:

$$\hat{\pi}_{i1} - \tilde{\pi}_{i1} = \frac{\hat{x}_1(\alpha - 2\gamma\hat{x}_1)}{4} + \frac{\hat{x}_1(\alpha - \gamma\hat{x}_1)}{4}, \tag{B.24}$$

$$\hat{\pi}_{i1} - \tilde{\pi}_{i1} = \frac{\hat{x}_1(2\alpha - 3\gamma\hat{x}_1)}{4}, \quad where \;\; i = A, B. \tag{B.25}$$

From equation (B.25), it is evident that when $\hat{x}_1 > \frac{2\alpha}{3\gamma}$, equation (B.25) is negative that is $\hat{\pi}_{i1} < \tilde{\pi}_{i1}$ for both A, B. The SPNE in this case will be to not take revenge. When $\hat{x}_1 < \frac{2\alpha}{3\gamma}$, equation (B.25) is positive that is $\hat{\pi}_{i1} > \tilde{\pi}_{i1}$ for both A, B. The SPNE in this case will be to take revenge.

B.0.5 Simplification of equation (4.25)

We can write equation (4.20) in the following way;

$$\hat{x}_{A1} + \frac{2R_A^3 x_{A1}(\alpha_B - 2\gamma_B x_{A1})}{(R_A + R_B)^3} = \hat{x}_{B1} + \frac{2R_B^3 x_{B1}(\alpha_A - 2\gamma_A x_{B1})}{(R_A + R_B)^3}. \tag{B.26}$$

$$\hat{x}_{A1}(1 + \frac{2R_A^3(\alpha_B - 2\gamma_B x_{A1})}{(R_A + R_B)^3}) = \hat{x}_{B1}(1 + \frac{2R_B^3(\alpha_A - 2\gamma_A x_{B1})}{(R_A + R_B)^3}). \tag{B.27}$$

We know from the formulation of the revenge function that $R_i^{'} = (\alpha_i - 2\gamma_i x_{j1})$ where, $i \neq j$, thus we can write the above equation in the following way;

$$\hat{x}_{A1}(1 + \frac{2R_A^3 R_B^{'}}{(R_A + R_B)^3}) = \hat{x}_{B1}(1 + \frac{2R_B^3 R_A^{'}}{(R_A + R_B)^3}). \tag{B.28}$$

Appendix C

Appendix: Chapter 5

C.0.1 Complete proof of separating equilibrium A

Strong protesters (R_s): Reject & Protest.
Weak protesters (R_w): Accept.
Government(G): Beliefs over the protest $[R_s, R_w] = \{1, 0\}$. G believes the strong protesters (R_s) protest. The government(G) accommodates in the 'Reject & Protest' information set. G believes that weak protesters (R_w) do not protest and the game ends when there is no protest.

G's best response:

$$\tilde{\pi}_G < (1 - w_2)M \tag{C.1}$$

$$w_2 < \frac{3}{4} \tag{C.2}$$

R_s won't deviate that is it would protest when;

$$w_2M - c_s > w_0M \tag{C.3}$$

R_w won't deviate that is it won't protest when;

$$w_1M - c_w < w_0M \tag{C.4}$$

This separating PBE holds when;

$$w_2 < \frac{3}{4} \tag{C.5}$$
$$w_1 - \frac{c_w}{M} < w_0 < w_2 - \frac{c_s}{M} \tag{C.6}$$

C.0.2 Complete proof of separating equilibrium B

Strong protesters (R_s): Reject & Protest.
Weak protesters (R_w): Accept.
Government(G): Beliefs over the protest $[R_s, R_w] = \{1, 0\}$. G believes the strong protesters (R_s) protest. The government (G) represses in the 'Reject & Protest' information set. G believes that weak protesters (R_w) do not protest and the game ends when there is no protest.

G's best response:

$$\tilde{\pi}_G > (1 - w_2)M \tag{C.7}$$
$$w_2 > \frac{3}{4} \tag{C.8}$$

R_s won't deviate that is it would protest when;

$$\tilde{\pi}_R > w_0 M \tag{C.9}$$
$$\frac{M}{4} - c_s > w_0 M \tag{C.10}$$

R_w won't deviate that is it won't protest when;

$$\hat{\pi}_R < w_0 M \tag{C.11}$$
$$\frac{\gamma^2 M}{(1+\gamma)^2} - c_w < w_0 M \tag{C.12}$$

This separating PBE holds when;

$$w_2 > \frac{3}{4} \tag{C.13}$$

$$\frac{\gamma^2}{(1+\gamma)^2} - \frac{c_w}{M} < w_0 < \frac{1}{4} - \frac{c_s}{M} \tag{C.14}$$

C.0.3 Complete proof of separating equilibrium C

Strong protesters (R_s): Accept.
Weak protesters (R_w): Reject & Protest.
Government (G): Beliefs over the protest $[R_s, R_w] = \{0, 1\}$. G believes the weak protesters (R_w) protest. The government (G) accommodates in the 'Reject & Protest' information set. G believes that strong protesters (R_s) accept the proposal and the game ends when there is no protest.

$$\hat{\pi}_G < (1 - w_1)M \tag{C.15}$$

$$w_1 < 1 - \frac{1}{(1+\gamma)^2} \tag{C.16}$$

R_w won't deviate that is it would protest when;

$$w_1 M - c_w > w_0 M \tag{C.17}$$

R_s won't deviate that is it won't protest when;

$$w_2 M - c_w < w_0 M \tag{C.18}$$

This separating PBE holds when;

$$w_1 < 1 - \frac{1}{(1+\gamma)^2} \tag{C.19}$$

$$w_2 - \frac{c_s}{M} < w_0 < w_1 - \frac{c_w}{M} \tag{C.20}$$

Now, $(w_2 - w_1) + \frac{(c_w - c_s)}{M} > 0$, this implies that the lower limit of (C.20) is more than the upper limit, which is not possible. Thus this separating equilibrium cannot hold.

C.0.4 Complete proof of separating equilibrium D

Strong protesters (R_s): Accept.
Weak protesters (R_w): Reject & Protest.
Government(G): Beliefs over the protest $[R_s, R_w] = \{0, 1\}$. G believes the weak protesters (R_w) protest. The government (G) repress in the 'Reject & Protest' information set. G believes that strong protesters (R_s) accept the proposal and the game ends when there is no protest.

$$\hat{\pi}_G > (1 - w_1)M \tag{C.21}$$
$$w_1 > 1 - \frac{1}{(1+\gamma)^2} \tag{C.22}$$

R_w won't deviate that is it would protest when;

$$\hat{\pi}_R > w_0 M \tag{C.23}$$
$$\frac{\gamma^2 M}{(1+\gamma)^2} - c_w > w_0 M \tag{C.24}$$

R_s won't deviate that is it won't protest when;

$$\tilde{\pi}_R < w_0 M \tag{C.25}$$
$$\frac{M}{4} - c_s < w_0 M \tag{C.26}$$

This separating PBE holds when;

$$w_1 > 1 - \frac{1}{(1+\gamma)^2} \tag{C.27}$$
$$\frac{1}{4} - \frac{c_s}{M} < w_0 < \frac{\gamma^2}{(1+\gamma)^2} - \frac{c_w}{M} \tag{C.28}$$

Now, $(\frac{1}{4} - \frac{\gamma^2}{(1+\gamma)^2}) = \frac{1-3\gamma^2+2\gamma}{4(1+\gamma)^2}$ which can be written in the following way;

$(\frac{1}{4} - \frac{\gamma^2}{(1+\gamma)^2}) = \frac{(1-\gamma)(1+3\gamma)}{4(1+\gamma)^2} > 0$ since $0 < \gamma < 1$.

Thus, again the lower limit of (C.28) is more than the upper limit, which is not possible. Thus, this separating equilibrium cannot hold.

C.0.5 Sustainability of pooling equilibrium under intuitive criteria

Government (G) is better-off accommodating the weak protester (R_w) and repressing the strong protesters (R_s). G accommodates when, $\frac{\alpha_0}{(1-\alpha_0)} > \frac{\hat{\pi}_G-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G}$. Now, let us consider the sequential equilibrium where;
Protesters (both the types, R_s, R_w): Accept.
Government (G): Accommodates the protesters.
This sequential equilibrium will hold when $w_0M > w_2M - c_s > w_1M - c_w$. G accommodates the protesters, $\frac{\alpha_0}{(1-\alpha_0)} > \frac{\hat{\pi}_G-(1-w_1)M}{(1-w_2)M-\tilde{\pi}_G}$. Now, let us consider the case that the strong protesters are better-off under repression than the equilibrium payoff of acceptance, $\tilde{\pi}_R > w_0M$ and G is better-off repressing the strong protesters, $\tilde{\pi}_G > (1-w_2)M$. The equilibrium payoff dominates for the weak protesters (R_w). Now, if there is an off-the-equilibrium action of protest G will believe it is the strong protesters (R_s) and repress and R_s will deviate and protest. Now, this will happen when;

$$\frac{M}{4} - c_s > w_0M \tag{C.29}$$

$$w_0M > w_2M - c_s \tag{C.30}$$

From equations (C.29) and (C.30) we get;

$$(w_2 - w_0)M < c_s < (\frac{1}{4} - w_0)M \tag{C.31}$$

For the sustainability of equation (C.31) it must be $w_2 < \frac{1}{4}$. G will be better-off repressing the strong protesters (R_s) when;

$$(1 - w_2)M < \frac{M}{4} \implies w_2 > \frac{3}{4} \tag{C.32}$$

Equations (C.31) and (C.32) cannot hold together. Thus, this equilibrium will always sustain.

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