INDIAN STATISTICAL INSTITUTE
203 B.T. Road, Kolkata – 700108
M. Stat. 2nd year (2024 -25)
Semester II Examination
Financial Econometrics

09 May 2025 Maximum Marks: 100 Time: 3 hours

*[This question paper has two parts – PART-I and PART-II. Use separate answer scripts for the two parts. Answer any **FIVE** questions from PART-I and **ALL** questions from PART-II. Marks allotted to each question are given within parentheses].*

PART-I

1. Discuss the problem of endogeneity in the following stationary time series model.

   $y_t = \alpha y_{t-1} + \varepsilon_t, \ |\alpha| < 1$

   where ε_t follows a moving average process of order 2. Describe with derivations, if required, how you can obtain a consistent estimator of the parameter α. Will this estimator be efficient? Explain.

   [6 +6+2= 14]

2. (a) Specify the random effects model along with all assumptions.
   (b) Obtain consistent and efficient estimator of the regression coefficients of this model.

   [5 + 9 = 14]

3. (a) Discuss why the fixed effects model is considered to be useful in analysing panel data. Also, state in what respects(s) this model is different from the random effects model.

   (b) Describe an estimation procedure for the parameters of this model. Is the estimator thus obtained consistent as well as efficient? Give justification in support of your answer. Discuss why this estimator is also called the least squares dummy variable estimator.

   [(4+2)+(4+2+2) = 14]

4. Suppose a panel data set having N cross sectional units and T time points is stacked in pooled form, ignoring the panel structure. Then a standard K-variable linear regression model where the intercept and slope parameters are assumed to be simultaneously homogeneous among different cross sectional units at different times, is considered. The errors of this model are assumed to be *iid* $N(0,\sigma^2)$ for all cross-sectional units and at all times.

   (a) Derive an unbiased estimator of σ^2 using this pooled data.

   (b) Suppose that you now want the intercept and slope parameters of this model to capture the cross sectional effects of the data. Describe a test procedure for testing all possible hypotheses of homogeneity involving the intercept and slope parameters; state all the relevant null hypotheses, the test statistics and their distributions in your answer.

[4 + 10 = 14]

5. (a) In a simultaneous equations system/model with G endogenous and K exogenous variables, discuss the problem of identification of the structural equations. What is the most common solution applied to overcome this problem? Give justification for your answer.

   (b) Derive the rank condition for identifiability of a structural equation of this simultaneous equations system.

[(5+2) + 7 = 14]

6. (a) The structure of a model with four endogenous and three predetermined variables is as follows (1 indicates presence and 0 absence of the variable in the equations).

$$\begin{matrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{matrix}$$

Write the structural equations of this four-equation system. Also, assess the identifiability of the parameters of these four structural equations by applying a suitable rank condition.

(b) Do you think that the ordinary least squares method of estimation produces consistent estimate of the coefficient parameters of an exactly identified structural equation of a simultaneous equations system? Give justification with derivations, if relevant, in support of your answer.

[(2+8) + 4 = 14]

PART-II

1. An ARCH (2) process in a dynamic linear model set-up is given as follows

$$y_t = x_t'\beta + u_t,$$

$$u_t = z_t\sqrt{h_t},\ z_t \sim iid\ N(0,1), \text{ and } u_t|\Psi_{t-1} \sim N(0, h_t),$$

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2$$

where x_t is a $k \times 1$ vector of exogenous variables, which may include lagged values of the dependent variable, β is a $k \times 1$ vector of regression parameters, and Ψ_{t-1} is the information set.

Find the covariance stationarity conditions of u_t^2 in terms of the parameters α_1 and α_2.

[14]

2. (a) Discuss what you mean by spurious regression in case of time series variables. What are the remedies for spurious regression? Explain.

(b)Show that in case of two variables, the number of cointegrating vector is one.

(c) Describe the Engle-Granger test for cointegration involving two variables.

[(4+4)+2+6 =16]