M.TECH./ JRF (CS)
Mid Semester Exam
Quantum Computation
26th February 2025

Total Marks: 50 Total time: 120 minutes

Question 1. (10 points) In the circuit below $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$. What is the quantum state of the three qubits at the other end of the circuit?

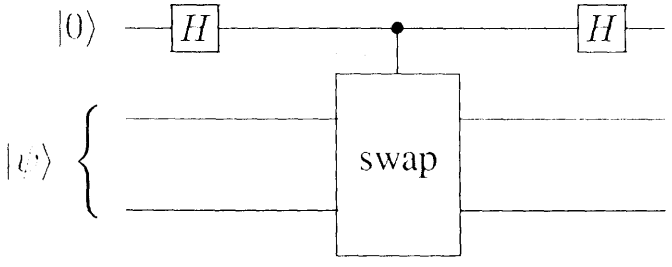


Figure 1

Question 2. (10 points) Consider the following circuit which is called the SWAP test in the literature. In the circuit below we perform the measurement with respect to $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$. Further, $\times$ in the circuit means the controlled SWAP gate (the one which was discussed in the assignment). Calculate $\Pr\{\text{Outcome} = 0\}$. Prove or disprove that this circuit can be used to estimate $|\langle\psi|\phi\rangle|^2$.

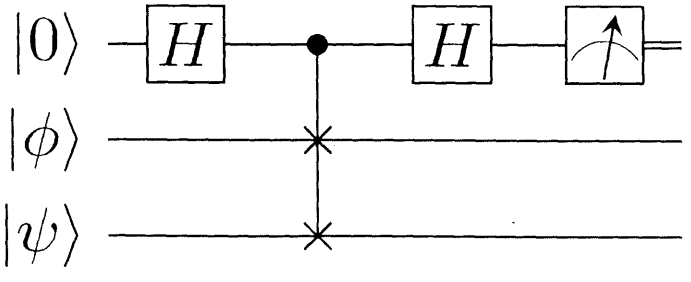


Figure 2

Question 3. (15 marks)

Alice and Charlie share a general two-qubit state $|\Psi\rangle^{A_1C} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$, of which Alice holds register A_1 and Charlie holds register C. Alice also shares the Bell state $|\phi^+\rangle^{A_2B}$ with

Bob, where Alice holds A_2, Bob holds B. Now Alice and Bob perform the teleportation protocol to send the state in Alice's register A_1 to Bob, using their shared Bell state in A_2B. Show that after the teleportation protocol, Bob and Charlie share the state $|\Psi\rangle^{BC} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$.

(Note: You can swap the order of registers in Dirac notation as convenient. For example, it is fine if you find $|\Psi\rangle^{CB} = \alpha|00\rangle + \gamma|01\rangle + \beta|10\rangle + \delta|11\rangle$. But indicate clearly when you're doing this.)

Question 4. (15 marks) Suppose a function $f : \{0,1\}^2 \to \{0,1\}$ is given by a blackbox B_f in the usual way. That is,

$$B_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle.$$

where $|x\rangle$ are the input qubits and $|y\rangle$ is the answer qubit. Show that the quantum algorithm mentioned in Figure 3 computes

$$f(00) \oplus f(01) \oplus f(10) \oplus f(11)$$

with only two uses of the blackbox B_f.

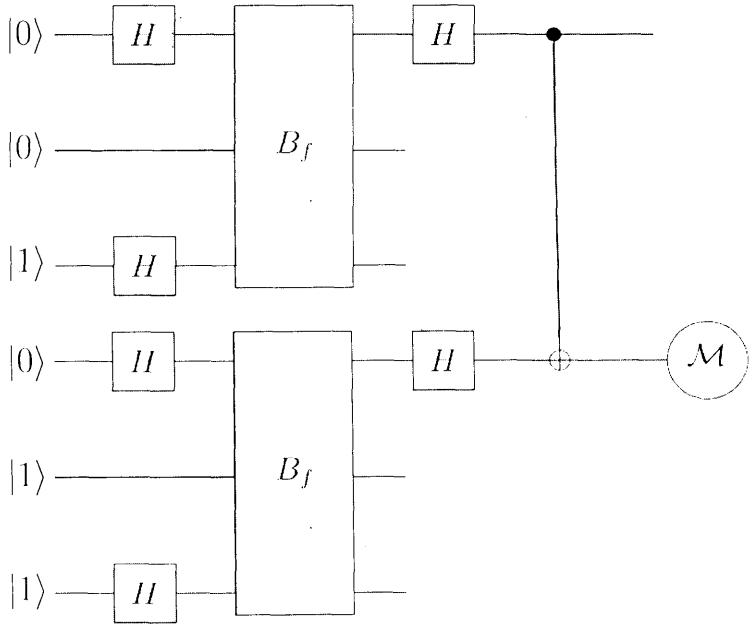


Figure 3

Hint: The whole set up is similar to the quantum algorithm which was discussed in the class to find u for $f(x) = u.x$, where $u, x \in \{0,1\}^n$.