INDIAN STATISTICAL INSTITUTE

M.Tech (CS) I Year: 2024-25 (Semester II)

Statistical Methods: Mid-Semester Exam

Date: 26/02/2025 Time: 2:30 PM - 4:30 PM Full Marks: 30

Note: Show necessary steps to solve all of the following problems. There is no credit for a solution if the appropriate work is not shown even if the answer is correct. The total marks allotted in this question paper is 32. However, if you score, say x marks out of 32 marks, your final score will be counted as $\min\{x, 30\}$ marks. Here, $n > 2$ is a natural number. Use of calculator is allowed in this exam.

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1. The following data represents interest rates charged by ten credit card companies.

   8.3, 7.9, 7.7, 8.4, 8.3, 8.7, 8.1, 7.8, 7.9, 9.4

   (a) Find the upper, lower fences and outliers (if any) in the dataset. [3]

   (b) Construct a box plot and comment on the skewness of the data. [3+1]

   (c) Suppose the above observations are values of a variable x. Find the Kendall's Tau between x and z where $z = 3 + \frac{1}{x} - 4x^5$. Justify your answer. [2]

2. Suppose $X_1, \ldots, X_n$ are i.i.d. copies of a random variable X with probability density function

   $$f(x \mid \alpha, \beta) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} \mathbb{I}_{\{0<x\leq\beta\}}$$

   (a) Find a two-dimensional sufficient statistic for (α, β). [2]

   (b) Derive the maximum likelihood estimator (MLE) of (α, β). [4]

   (c) Find the median m of the distribution of X and compute its MLE. [2]

3. Suppose $X_1, \ldots, X_n$ are i.i.d. observations having common density function

   $$f(x \mid \theta) = \frac{4\theta^4}{x^5} \mathbb{I}_{\{x\geq\theta\}}, \text{ where } \theta > 0.$$

   (a) Find the method of moment estimator of θ, say $\widehat{\theta}$. [2]

   (b) If $X_{(1)} = \min\{X_1, \ldots, X_n\}$, then show that $\delta(\mathbf{X}) = \frac{4n-1}{4n} X_{(1)}$ is both unbiased and consistent estimator of θ. [2+2]

   (c) What is the efficiency of $\widehat{\theta}$ relative to $\delta(\mathbf{X})$? [4]

4. Suppose X_1 and X_2 are i.i.d. *Bernoulli*(θ) with probability mass function $p(x|\theta) = \theta^x(1-\theta)^{1-x}$ for $x \in \{0, 1\}$ and $\theta \in (0, 1)$. If $T = 3X_1 - 2X_2$, then argue why the Fisher information $I_T(\theta)$ in T must be less than or equal to $\frac{2}{\theta(1-\theta)}$. State the result you may need to use. [4+1]