Indian Statistical Institute, Kolkata

Midterm examination: Second semester 2024 - '25

B.Stat I year

Subject	:	**Analysis II**
Time	:	2 hours
Maximum score	:	30

Work independently. Attempt all the problems. Please use a new page to answer each problem and make sure that the problem number in the margin can be read, even after stapling. If you attempt the same problem several times, please strike out all the attempts except the correct one before submitting your answer script. Justify every step in order to get full credit of your answers. All arguments should be clearly mentioned on the answer script. Points will be deducted for missing or incomplete arguments.

(1) Find $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+k^2}}$ by interpreting this as a Riemann sum corresponding to an integral.

[4 marks]

(2) (i) Let $f, g : [a, b] \to \mathbb{R}$ be two continuous functions such that $\int_a^b f(x)dx = \int_a^b g(x)dx$. Prove that there exists a point $c \in [a, b]$ such that $f(c) = g(c)$.

(ii) f is a continuous function on $\mathbb{R}$ such that $\int_{-x}^{x} f(t)dt = 0$ for all $x \in \mathbb{R}$. Prove that f is an odd function.

[4 + 4 = 8 marks]

(3) Let $f_n(x) = n + \frac{x}{n}, x \in \mathbb{R}$. Show that

(i) the sequence $\{f_n'\}$ is uniformly convergent on $\mathbb{R}$,

(ii) the sequence $\{f_n\}$ is not uniformly convergent on $\mathbb{R}$. Explain why the sequence $\{f_n\}$ is not uniformly convergent on $\mathbb{R}$ although (i) is satisfied.

[2 + 3 = 5 marks]

(4) Evaluate the following improper integrals:

(i) $\displaystyle\int_0^\infty \frac{\sin^4 x}{x^2} dx$. (You may assume $\int_0^\infty \frac{\sin x}{x} dx = \pi/2$).

(ii) $\displaystyle\int_0^\infty \frac{\sin x}{\sqrt{x}} dx$.

[4 + 4 = 8 marks]

(5) Find $\lim_{x\to 0} \sum_{n=1}^{\infty} \frac{\cos(nx)}{n(n+1)}$

[5 marks]