

Indian Statistical Institute
Probability Theory II

*B.Stat (hons) 1st year,
Semestral examination*

Date: May 02, 2025

Duration: 3 hrs.

Attempt all questions. The maximum you can score is 50. Justify all your steps. This is a closed book, closed notes examination. It is extremely important that you clearly write the results you are using. Vague scribbles, contradictory arguments will not be given any credit.

1. Prove or disprove, as appropriate, each of the following statements.

(a) The Cantor distribution has finite MGF (say, $m(t)$) for all $t \in \mathbb{R}$.

[5]

(b) For a random variable X taking value in $[0, 1]$ the maximum possible value for $V(X)$ is $\frac{1}{12}$.

[5]

2. If X, Y are independent $N(0, 1)$ random variables, then find the density of $R = \sqrt{X^2 + Y^2}$.

[10]

3. Let $\mathbf{Y} \sim N_n(\mathbf{A}\mathbf{b}, I)$ where $A_{n \times p}$ and $\mathbf{b}_{p \times 1}$ are fixed. Let $\mathbf{Z}$ be the orthogonal projection of $\mathbf{Y}$ on the column space of A . Then find the distribution of $\|\mathbf{Y} - \mathbf{Z}\|^2$.

[10]

4. Suppose that the travel time from your home to your office is normally distributed with mean 40 minutes and standard deviation 7 minutes. If you want to be 95% certain that you will not be late for an office appointment at 1 pm, what is the latest time you should leave home? [You may leave your answer in terms of the standard normal CDF $\Phi(\cdot)$ and/or its inverse.]

[10]

5. Let (X, Y) have joint probability density function proportional to $\exp(-2x^2 - 8y^2 + 2xy + 4x)$ for $x, y \in \mathbb{R}$. Find $E(X)$, $E(Y)$ and $\text{corr}(X, Y)$.

[2+2+6]

6. If $(X_1, X_2) \sim \text{Dirichlet}(a_1, a_2, a_3)$, then show that $X_1 \sim \text{Beta}(a_1, a_2 + a_3)$.

[5]