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THE MEASUREMENT OF SOCIAL EXCLUSION

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This paper develops an axiomatic approach to the measurement of social exclusion. At the individual level, social exclusion is viewed in terms of deprivation of the person concerned with respect to different functionings in the society. At the aggregate level we treat social exclusion as a function of individual exclusions. The class of subgroup decomposable social exclusion measures using a set of independent axioms is identified. We then look at the problem of ranking exclusion profiles by the exclusion dominance principle under certain restrictions. Finally, applications of decomposable and non-decomposable measures suggested in the paper using European Union and Italian data are also considered.

1. INTRODUCTION

The subject of this paper is the measurement of social exclusion. The broad questions that we try to address in this paper are: (i) When do we say that an individual is socially excluded? (ii) What is the level of social exclusion in a country? (iii) Can we say that social exclusion in country A is less than that in country B? (iv) Given the level of social exclusion in a society, which subgroups of the population, partitioned according to ethnic, geographic, or any other socio-economic characteristic, contribute more to aggregate social exclusion? (v) When can we say that one society dominates another with respect to social exclusion and what are the consequences of such a dominance relationship?

Broadly speaking, a person is said to be socially excluded if he is unable to "participate in the basic economic and social activities of the society in which he lives." In the European Commission's Programme specification for "targeted socioeconomic research," social exclusion is described as "disintegration and fragmentation of social relations and hence a loss of social cohesion. For individuals in

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particular groups, social exclusion represents a progressive process of marginalization leading to economic deprivation and various forms of social and cultural disadvantage."

As Atkinson (1998) said, social exclusion is not just a consequence of unemployment. It is true that an unemployed person may not have income to maintain a subsistence standard of living and hence becomes socially excluded. But many employed persons may not be integrated fully in the society they live in. Expansion of employment may increase the income gap between low-paid and high-paid workers and hence it may not reduce or end social exclusion. Social exclusion may arise from the operations of the market and supplies of key goods and services. For instance, people may not be able to participate in the customary consumption activities because profit maximizing prices may exclude them from the markets. A person may not be allowed to have an account in a bank if he does not fulfil certain constraints. It can as well emanate from operations of the State if the State's social security benefit programmes are targeted towards some particular groups or persons.

As social exclusion includes economic, social and political aspects of life, it is a *multidimensional* phenomenon. Since fundamental to achieving human choices is building human capabilities, we can also interpret the issue in terms of (i) functionings, the various things a person value doing or being and (ii) capability, the ability to achieve (Sen, 1985). The valued functionings may vary from such elementary ones as adequate nourishment and literacy, to complex activities like participation in social gatherings and having self respect. The standard of living in this framework is determined by the opportunity set of basic capabilities to function. The freedom of choice, that is, the extent of opportunities available rather than merely the point chosen becomes an important component of living standard. Now, if social exclusion is viewed as the inability to meet needs valuable to the individual, then regarding it as capability failure makes considerable sense. We regard the concept of capability failure as a notion of deprivation because people feel deprived when they lack such opportunities.[1] Hence social exclusion implies deprivation in a wide range of indicators or functionings of living standards, which can be of quantitative or qualitative type.[2]

Social exclusion is related to both inequality and poverty, but should not be equated with either of them (Atkinson, 1998). According to Sen (1998), social exclusion is wider than poverty. Multidimensional inequality is a measure of the dispersion of the multidimensional distribution of quantities of consumption of the functionings for different individuals (Tsui, 1999). Multidimensional poverty measurement, on the other hand, specifies a poverty threshold for each functioning, looks at the shortfalls of the functioning quantities of different individuals from the threshold levels, and aggregates these shortfalls into an overall magnitude of poverty (Bourguignon and Chakravarty, 2003). Thus, both multidimensional poverty and social exclusion deal with capability failures, while in the former we view it in terms of the shortfalls from thresholds in a given point in time, in the

[1]See Runciman (1966) for a general treatment of deprivation.

[2]See Atkinson *et al.* (2002) for a list of functionings that can be used for the measurement of social exclusion.

latter the problem is one of inability to participate.[3] Note further that in the case of both multidimensional inequality and poverty the functionings have to be of quantitative type, whereas social exclusion considers qualitative type functionings as well. Social exclusion can be regarded as a state and as a process leading to deprivation in the form of non-participation. More explicit differences may be noted. A country with low deprivation (in terms of non-participation) but high degree of dispersion among attribute quantities and high levels of shortfalls of meagre attribute sizes from respective thresholds will be characterized with high inequality and high poverty but low exclusion. Similarly, there may be situations with high exclusion but low inequality and poverty.

Atkinson (1998) argued further that it is a *relative* concept, we cannot say whether a person is socially excluded or not by looking at his position alone. The positions of the others in the society have to be taken into account for a proper implementation of any criterion for exclusion. It has, furthermore, a *dynamic* character because an individual is socially excluded if his deprivation continues or worsens over time.

Three types of implicit conceptualization of social exclusion are currently available in the literature. In the first, it is interpreted as the lack of participation in social institutions (Duffy, 1995; Rowntree Foundation, 1998; U.K. House of Commons, 1999; Paugam and Russell, 2000); whereas the second regards the problem as the denial or non-realization of rights of citizenship (Room, 1995; Klasen, 2002). Finally, the third views social exclusion in terms of increase in distance among population groups (Akerlof, 1997; Bossert, D'Ambrosio and Peragine (BDP), 2004). Some researchers attempted to suggest measures of social exclusion building on these approaches (see, among others, Bradshaw *et al.*, 2000; Tsakloglou and Papadopoulos, 2002). However the theoretical foundations of these measures are often unclear.

In this paper we adopt an axiomatic approach to the measurement of social exclusion. An alternative approach has been proposed by BDP. To the best of our knowledge, theirs is the only other axiom-based paper. The two contributions exhibit substantial differences in how different aspects of social exclusion are taken into consideration (see Section 2 for details).

Since in order to be socially integrated a person needs to have access to some social functionings, we first look at the capability failure, that is, the number of functionings from which the person is excluded over time. This number may be regarded as the deprivation score of the person under consideration. However, some of the functionings may be more important than others. Therefore, a more general way is to assign an integer weight to each failure depending on the importance of the functioning and the deprivation score of a person is the sum of these integers.

[3]Tsakloglou and Papadopoulos (2002) proposed an index of social exclusion based on the distribution of an individual welfare indicator. Imposing a threshold, they identified a person at high risk of deprivation if his indicator falls below the threshold. The dynamic aspect of social exclusion is included by considering the number of years during which the deprivation takes place. Evidently, specification of such a threshold involves some degree of arbitrariness. Since our approach does not rely on a threshold of this type, it has an advantage over that of Tsakloglou and Papadodopoulos (2002).

The social exclusion measure that we propose is a real valued function of the deprivation scores of different individuals in the society. In a sense our approach is similar to the view that considers social exclusion as lack of participation in social institutions, where lack of participation is treated as capability failures. We first characterize the family of exclusion measures whose members satisfy *normalization*, *monotonicity*, *subgroup decomposability*, and have *nondecreasing marginals*.

Normalization means that social exclusion is zero if nobody is socially excluded. Monotonicity requires the measure to increase if the deprivation score of a person increases. According to subgroup decomposability, for any partitioning of the population with respect to some socioeconomic or demographic characteristic, the overall social exclusion is the population share weighted average of subgroup exclusion levels. This property enables us to calculate a particular subgroup's contribution to aggregate exclusion and hence to identify the subgroups that are more afflicted by exclusion and to implement anti-exclusion policy. Clearly, according to this notion of policy recommendation, an assessment of overall exclusion becomes contingent on the implicit valuation of the exclusion measure. However, an exercise of this type may be useful for two reasons. First, following Sen (1985), the nonwelfarist approach to policy analysis is becoming quite popular. Second, in many situations policy is evaluated using specific forms of measures. So it seems worthwhile to see what type of policy would be implied by the use of a specific exclusion measure.

Marginal social exclusion is defined as the change in social exclusion when we increase the deprivation score of a person by one. Nondecreasingness of marginal social exclusion ensures that in aggregating individual deprivation scores into an overall indicator of exclusion, a higher deprivation score does not get a lower weight than a lower score.

The characterized family of measures is shown to possess some additional interesting properties. It is also shown that the properties employed in the characterization exercise are independent, that is, none of these properties implies or is implied by another.

In subgroup decomposability we calculate each subgroup's exclusion independently of exclusions of other subgroups. Thus, one subgroup's exclusion does not affect exclusions of other subgroups. We, therefore, have to use weights for different functionings that do not violate this condition. Hence the weights should be independent of the overall population size. However, an alternative assumption, which appears to be quite realistic, is dependence of weights on the population size (see Section 5 for one such approach). Consequently, it may also be worthwhile to study non-subgroup decomposable measures. We therefore consider two measures of this type, the symmetric mean exclusion of order $\nu > 1$ and the Gini exclusion measure, and use population size dependent weights to calculate them. These measures satisfy all the axioms except subgroup decomposability.

Next, we consider the problem of ranking two societies by the social exclusion dominance criterion. We demonstrate that for two societies with a common population size and the same total deprivation score, if one dominates the other by the exclusion dominance criterion, then the former becomes at least as socially excluded as the latter by all additive social exclusion measures that satisfy anonymity and have nondecreasing marginals. This result parallels the if part of

the well-known Atkinson (1970) result on Lorenz Domination which says that if u and v are two income distributions of a given total over a fixed population size, and if u Lorenz dominates v, then all symmetric utilitarian social welfare functions regard u at least as good as v, where the identical individual utility function is concave.

Finally, we apply different measures to the EU member states and to the Italian regions in the 1990s and consider some policy implications.

The paper is organized as follows. The next section introduces the formal framework for measuring social exclusion and presents the properties for an exclusion measure. In Section 3 we characterize the family of exclusion measures and discuss its properties. Section 4 deals with social exclusion dominance relation. The application is contained in Section 5. Section 6 concludes.

2. Properties for a Measure of Social Exclusion

Let $\mathbf{N}$ ($\mathbf{N}_0$) be the set of all positive (nonnegative) integers and $\mathbf{R}$ be the set of real numbers. For all $n \in \mathbf{N}$, D^n is the n-fold Cartesian product of $\mathbf{N}_0$ and 1^n is the n-coordinated vector of ones. For any society with a population of size $n \in \mathbf{N}$, there is a finite nonempty set of functionings F relevant for social integration. Throughout this paper we assume that F is fixed so that cross-population comparisons of social exclusion can be made in terms of elements of F.[4] An individual in an n-person society can be excluded from any subset of F, where $n \in \mathbf{N}$ is arbitrary. The degree of exclusion or deprivation of a person can be captured using the number of functionings from which he is excluded. For each functioning, we define a characteristic function which takes on the value 1 or 0 according as the person is excluded or not from the functioning. Since some functionings may be more important than others, the characteristic function of each functioning is weighted by an integer, where the integer weights are determined in terms of importance of the functionings.[5] The *deprivation score* of the person concerned is then given by the sum of integer weighted characteristic functions. More precisely, let $F_i \subseteq F$ be the set of functionings from which person i is excluded. Denote the weight attached to attribute j by w_j, then $x_i = \Sigma_{j \in Fi}\, w_j$. Note that this particular method of calculating deprivation is applicable to both qualitative and quantitative attributes.

This procedure of calculating the individual deprivation scores is quite similar to the Basu–Foster (1998) way of determining a household literacy profile. They assumed that individual literacy is a 0–1 variable and an adult member of a household is identified by the number 0 or 1 according to whether he is illiterate or literate. The total number of literates in the household is then simply the sum of the 1's in the household. This procedure can also be extended to the situation when literacy is assumed to be multidimensional.

We assume that the calculation of the deprivation score of person i, x_i, involves a dynamic or longitudinal aspect and depends on the rest of the society.[6] If x_i is positive, a trade-off between excluded and non-excluded functionings is not

[4]See Atkinson *et al.* (2002) for common elements of F for the EU as a whole.
[5]See Section 5 for one approach to the calculation of weights.
[6]See Section 5 for one example of the inclusion of dynamic considerations.

allowed. For instance, a person's high income cannot compensate the dissatisfaction associated with his job.

An *exclusion profile* in a society of n persons is a vector $x = (x_1, \ldots, x_n)$, where $x_i \in \mathbf{N}_0$ is the deprivation score of person i. The set of exclusion profiles for an n-person population is D^n, $n \geq 1$. Thus, $x \in D^n$ for some $n \in \mathbf{N}$. The set of all possible exclusion profiles is $D = \mathbf{U}_{n \in \mathbf{N}} D^n$.

A *measure of social exclusion* is a function E: $D \rightarrow \mathbf{R}$. For any $n \in \mathbf{N}$, the restriction of E on D^n is given by E^n. For any $n \in \mathbf{N}$, $x \in D^n$, $E^n(x)$ is a measure of the extent to which different individuals are excluded from the activities taking place in the society, that is, the degree of exclusion suffered by all individuals in the society as a whole. For all $n \in \mathbf{N}$, $x \in D^n$, let $S(x)$ be the set of persons with positive deprivation scores, that is $S(x) = \{i, 1 \leq i \leq n | x_i > 0\}$. For any $n \in \mathbf{N}$, $x \in D^n$, let q be the cardinality of $S(x)$, that is the number of persons in $S(x)$. For any $n \in \mathbf{N}$, $x \in D^n$, we write $\bar{x}$ for nonincreasingly ordered permutation of x, that is $\bar{x}_1 \geq \bar{x}_2 \geq \ldots \geq \bar{x}_n$.

We assume that an arbitrary exclusion measure $E : D \rightarrow \mathbf{R}$ should satisfy the following postulates.

Axiom 1: Normalization (NOM). For all $n \in \mathbf{N}$, $E^n(01^n) = 0$.

Axiom 2: Monotonicity (MON). For any $n \in \mathbf{N}$, $x \in D^n$ and for any i, $1 \leq i \leq n$,

$$E^n(x) < E^n(x_1, \ldots, x_{i-1}, x_i + c, x_{i+1}, \ldots, x_n),$$

where $c \in \mathbf{N}$.

Axiom 3: Nondecreasingness of Marginal Social Exclusion (NMS). For any $n \in \mathbf{N}$, $x \in D^n$, and for any i, j, $1 \leq i, j \leq n$, if $x_i \geq x_j$ then:

$$E^n(x_1, \ldots, x_{i-1}, x_i + 1, x_{i+1}, \ldots, x_{j-1}, x_j, x_{j+1}, \ldots, x_n) - E^n(x) \geq E^n(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_{j-1}, x_j + 1, x_{j+1}, \ldots, x_n) - E^n(x).$$

Axiom 4: Subgroup Decomposability (SUD). For any $x^i \in D^{n_i}$, $i = 1, \ldots, k$, $E^n(x) = \sum_{i=1}^{k} \frac{n_i}{n} E^{n_i}(x^i)$, where $x = (x^1, x^2, \ldots, x^k)$.

Axiom 5: Anonymity (ANY). For all $n \in \mathbf{N}$, $x \in D^n$, $E^n(x) = E^n(xP)$, where P is any $n \times n$ permutation matrix.[7]

Normalization is a miniminality principle. It says that if nobody is excluded from any functioning in the society, then the value of the social exclusion measure is zero. *NOM* has a relative flavor because it is based on an identical position of all

[7]An $n \times n$ matrix is a permutation matrix if each of its entries is either zero or one, and each of its rows and columns sums to one.

persons in the society. Monotonicity says that if the deprivation score of an individual increases, then social exclusion should increase. This axiom has a flavor similar to Sen's (1976) monotonicity axiom, which requires poverty to increase if the income deprivation of a poor person goes up. (See Bourguignon and Chakravarty, 2003, for a multidimensional analogue to Sen's axiom.) Now, in terms of capability failure curtailment of freedom of choice or opportunity of some persons can certainly make them worse off given that the positions of all other persons remain unaffected. For instance, the lack of proper medical care for some persons and for all persons are possibly two situations of exclusion, the latter being more severe than the former (see Sen, 1985; Xu, 2002). The axiom *MON* tries to capture this idea. Evidently, social exclusion is a multifaceted phenomenon and we try to look at the problem as one of capability failure. But there can also be other views concerning its measurement and in such cases *MON* may not be a relevant postulate (see, for example, BDP). If a social exclusion measure satisfies *NOM* and *MON*, then it will take a positive value if at least one individual has a positive deprivation score.

Sen (1976) argued that in income poverty measurement the poverty line can be taken as the reference point for all poor persons and the poverty gap of a poor person, his income shortfall from the poverty line, is a measure of deprivation suffered by him. In order to attach higher weight to higher deprivation, Sen assumed that the weight on individual i's poverty gap is equal to his rank in the income distribution of the poor. This guarantees that an increase in poverty due to a reduction in the income (increase in deprivation) of the poor will be higher the lower (higher) is the income (deprivation) of the poor. Conversely, in order that an increase in poverty due to reduction in the income of the poor is higher the lower the income of the poor is, a necessary condition is to attach higher weight lower down the income scale. Our *NMS* postulate has a similar spirit. We consider two persons where the deprivation score of the first is not lower than that of the second. Then the change in social exclusion, if the deprivation score of the former increases by one, is at least as large as the corresponding change when the deprivation score of the latter increases by the same amount. Since *NMS* affects deprivations of two persons directly, it also reflects that social exclusion is a relative phenomenon.

SUD, which expresses aggregate exclusion in a society as a weighted average of subgroup exclusion levels, where the weights are population shares of the subgroups, is very important from policy point of view. $\frac{n_i}{n} E^{n_i}(x^i)$ is the contribution of subgroup i to total exclusion, i.e. the amount by which social exclusion will decrease if exclusion in subgroup i is eliminated. $\left(\frac{n_i E^{n_i}(x^i)}{nE^n(x)}\right) 100$ is the percentage contribution of subgroup i to total exclusion. Each of these figures is useful to planners and analysts to formulate anti-exclusion policies. It may be important to note that if x_i's are dependent on the population size, *SUD* may be violated.

Finally, *ANY* means that the exclusion measure is symmetric, i.e. any reordering of the deprivation scores leaves the exclusion level unchanged. *ANY* is unavoidable as long as the individuals are not distinguished by anything other than deprivation scores.

An interesting implication of *SUD* and *ANY* is the principle of population, which requires social exclusion to remain unaltered under any $m(\geq 2)$-fold replication of population (see Chakravarty and Majumder, 2006). This principle allows us to make cross-population comparisons of social exclusion.

Since to the best of our knowledge, the only other axiom-based paper in this area is by BDP, it seems worthwhile to compare our approach with the alternative approach of BDP who argued that social exclusion can be interpreted as persistence in the state of deprivation. At the outset BDP characterized measures of individual deprivation, which have been sequentially transformed into measures of social exclusion. While in the present paper it is assumed that minimal level of social exclusion is achieved when nobody is excluded from any functioning, in the BDP framework minimal value of individual deprivation is reached if everybody has the same number of capability failures, however small or large it may be. Further, their measures are homogeneous of degree one and satisfy translation invariance, where translation invariance of a measure requires it to remain unchanged under equal absolute changes in all failures. Two "proportionality properties" defined in terms of replications of the population, a conditional "anonymity" principle, which is different from ours, and a "focus axiom" which says that a person's deprivation depends on his capability failures and on those of individuals who have fewer failures, have also been used in the characterization exercise. Their measures of social exclusion are not subgroup decomposable. In view of this discussion, it is clear that the two approaches are quite different.

3. The Family of Subgroup Decomposable Social Exclusion Measures

In this section we derive the class of social exclusion measures whose members satisfy *NOM*, *MON*, *NMS*, in addition to *SUD*. Let Φ be the class of all functions $f: \mathbf{N}_0 \to \mathbf{R}$ such that $f(0) = 0$, f is increasing, and f has a nondecreasing marginal, that is:

$$f(x_i+1)-f(x_i) \geq f(x_j+1)-f(x_j), \tag{1}$$

where $x_i \geq x_j$.

For theorems 1 and 2 of this section we assume that the weights attached to different functionings are independent of the population size.

We then have:

Theorem 1: A social exclusion measure $E: D \to \mathbf{R}$ *satisfies NOM, MON, NMS, and SUD if and only if for all* $n \in \mathbf{N}$, $x \in D^n$,

$$E^n(x) = \frac{1}{n} \sum_{i \in S(x)} f(x_i), \tag{2}$$

where f is a member of Φ.

Proof: Let $n \in \mathrm{N}$ and $x \in D^n$ be arbitrary. Then by repeated applications of *SUD*:

$$E^n(x) = \frac{1}{n}\sum_{i=1}^{n} E^1(x_i). \tag{3}$$

We can rewrite E^n in (3) as:

$$E^n(x) = \frac{1}{n}\sum_{i=1}^{n} f(x_i), \tag{4}$$

where $f = E^1$. Clearly, $f : \mathbf{N}_0 \to \mathbf{R}$. *MON* demands increasingness of f. Now, suppose $x_i \geq x_j$. The inequality:

$$E^n(x_1, \ldots, x_{i-1}, x_i + 1, x_{i+1}, \ldots, x_{j-1}, x_j, x_{j+1}, \ldots, x_n) - E^n(x) \geq$$
$$E^n(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_{j-1}, x_j + 1, x_{j+1}, \ldots, x_n) - E^n(x),$$

on simplification, reduces to:

$$f(x_i + 1) - f(x_i) \geq f(x_j + 1) - f(x_j),$$

which is nondecreasingness of marginal of f. Clearly, if $x_i = 0$ for all i, then *NOM* requires that $f(0) = 0$.

Obviously, $f(0) = 0$ enables us to rewrite $\frac{1}{n}\sum_{i=1}^{n} f(x_i)$ as $\frac{1}{n}\sum_{i \in S(x)} f(x_i)$. This establishes the necessity part of the theorem on D^n for a given $n \in \mathbf{N}$.

The sufficiency is easy to verify. Since $n \in \mathbf{N}$ was chosen arbitrarily, our result holds for all $n \in \mathbf{N}$.

Note that the general measure in (2) satisfies *ANY* although we did not use these properties in its derivation. We can interpret f in (2) as the individual exclusion function. An alternative way of writing the formula (2) is:

$$E^n(x) = \frac{H}{q}\sum_{i \in S(x)} f(x_i), \tag{5}$$

where $H = \frac{q}{n}$ is the head-count measure of social exclusion, the proportion of persons that is socially excluded in the population. For a fixed n, on social exclusion profiles with a given q, H is a constant function. Thus H is violator of *MON* although it meets *NOM*, *SUD*, *ANY*, and *NMS*.

The head-count measure of social exclusion is quite analogous to the multidimensional poverty head-count ratio. Multidimensional poverty measurement considers for each person a poverty indicator variable that takes on the value of 1 if his consumption of some attribute(s) falls below the corresponding threshold(s). Otherwise the indicator variable assumes the value zero. The total number of multidimensional poor is then given by the sum of indicator variables across persons (see Bourguignon and Chakravarty, 2003).

In order to illustrate the general formula in (2), let $f \in \Phi$ be of the form $f(t) = t^{\delta}$, $\delta \geq 1$. Then the corresponding measure is:

$$E_{\delta}^{n}(x) = \frac{H}{q} \sum_{i \in S(x)} x_i^{\delta}. \tag{6}$$

For any $\delta \geq 1$, E_{δ}^{n} satisfies all the postulates. For $0 < \delta < 1$, E_{δ}^{n} is a violator of *NMS* but not of others. As $\delta \to 0$, $E_{\delta}^{n} \to H$. The single parameter δ in (6) is a value judgement parameter. E_{δ}^{n} becomes more sensitive to the higher deprivation scores as δ increases from 1 to plus infinity. For a given $x \in D^n$, an increase in the value of δ does not decrease E_{δ}^{n}. For $\delta = 1$, E_{δ}^{n} becomes the average deprivation score of the society, that is, $A = \frac{1}{n} \sum_{i \in S(x)} x_i$. For $\delta = 2$, we can rewrite E_{δ}^{n} as:

$$E_{\delta}^{n}(x) = \sigma^2(x) + A^2(x), \tag{7}$$

where σ^2 is the variance of the society deprivation scores. Given A, a reduction in σ^2 reduces the measure in (7). Such a situation may arise if a higher deprivation score decreases and a lower deprivation score increases by the same amount. Over social exclusion profiles with the same population size and the same average deprivation score, the ranking of the profiles generated by E_{δ}^{n} (for $\delta = 2$) is the same as that generated by σ^2.

An alternative of interest arises from the specification $f(t) = e^{\alpha t} - 1$, where $\alpha > 0$. The resulting measure is:

$$E_{\alpha}^{n}(x) = \frac{H}{q} \sum_{i \in S(x)} \left(e^{\alpha x_i} - 1\right). \tag{8}$$

For a given $x \in D^n$, E_{α}^{n} is nondecreasing in α. E_{α}^{n} satisfies all the properties for all positive α. As α increases, the underlying evaluation attaches more weight to the higher deprivation scores.

We will now show that the postulates *NOM*, *MON*, *NMS* and *SUD* are independent. Independence means that none of the postulates implies or is implied by another, that is, none of them is redundant. It is thus a minimal condition. Therefore, if one of the postulates is dropped, there will be measures that will satisfy the remaining postulates but not the dropped one.

Theorem 2: The properties NOM, MON, NMS, and SUD are independent.

Proof:

(a) Evidently the measure $\breve{E}^{n}(x) = \frac{1}{n} \sum_{i \in S(x)} e^{x_i}$ is not normalized, but it will fulfil the other properties.

(b) Since the measure $\hat{E}^{n}(x) = -\frac{1}{n} \sum_{i \in S(x)} \frac{x_i}{1 + x_i}$ is decreasing in x_i, it is a violator of *MON*, but not of the remaining postulates.

(c) The measure $\tilde{E}^n(x) = \frac{1}{n}\sum_{i \in S(x)} x_i^\theta$, $0 < \theta < 1$, has a decreasing marginal and hence it fails to satisfy NMS, but it verifies the other properties.

(d) Since the measures $\bar{E}^n(x) = \left(\frac{1}{n}\sum_{i \in S(x)} x_i^\nu\right)^{\frac{1}{\nu}}$, $\nu > 1$, and $\hat{E}^n(x) = \frac{1}{n^2}\sum_{i \in S(x)} \bar{x}_i(2(n-i)+1)$ are not additive across components, they are not subgroup decomposable. However, they are normalized, monotonic, and have increasing marginals.

The measure $\bar{E}^n(x)$ is the symmetric mean exclusion of order $\nu\,(>1)$. We can refer to $\hat{E}^n(x)$ as the Gini exclusion measure since it involves a Gini type averaging.[8] One of our main objectives is certainly to calculate the additive measures, which demand weights to be independent of the overall population size. Alternatively, when dependence of weights on the population size is preferred, the two measures, $\bar{E}^n(x)$ and $\hat{E}^n(x)$, which satisfy all properties except *SUD*, could be used. In the empirical applications we will, therefore, show results for $E^n_\delta(x)$ in (6), $\bar{E}^n(x)$, and $\hat{E}^n(x)$.

It is clear that to every individual exclusion function $f \in \Phi$, there corresponds a different social exclusion measure of the form (2). They will differ only in the manner how a person's individual exclusion is specified as a function of his deprivation score. However, there is no guarantee that these social exclusion measures will rank exclusion profiles in the same way. We consider the problem of ranking exclusion profiles in the next section.

4. The Social Exclusion Dominance Relation

We begin this section by defining the social exclusion dominance criterion and look at its implications for exclusion profiles with a fixed total over a given population size.

For $x, y \in D^n$, we say that x dominates y by the social exclusion relation, which we write $x \succeq_{SE} y$, if:

$$\sum_{j=1}^{k} \bar{x}_j \geq \sum_{j=1}^{k} \bar{y}_j, \tag{9}$$

for all $k = 1, 2, \ldots, n$.

Given that the exclusion profiles $\bar{x}$ and $\bar{y}$ are ranked in nonincreasing order of capability failures of the individuals, $x \succeq_{SE} y$ demands that the cumulative deprivation score of the first k persons in $\bar{x}$ is at least as large as that in $\bar{y}$, where $k = 1, 2, \ldots, n$.

[8]Strictly speaking, when incomes are arranged in non-increasing order, the Gini index of inequality can be written as a linear function with weights being the odd natural numbers in increasing order. Since the averaging in $\hat{E}^n$ is quite similar in nature, we call it the Gini social exclusion measure.

In order to study implications of the relation $\succeq_{SE}$ in terms of exclusion measures, we now have the following:

Definition 1: For any $x \in D^n$, we say that $\bar{y}$ is obtained from $\bar{x}$ by a favourable composite change (FCC) if:

(10)
$$\begin{aligned} \bar{y}_i &= \bar{x}_i - 1 \\ \bar{y}_j &= \bar{x}_j + 1 \\ \bar{y}_k &= \bar{x}_k \quad \textit{for all } k \neq i, j, \end{aligned}$$

where $\bar{x}_i > \bar{x}_j$.

In *FCC* the degree of exclusion of a more deprived person (i) is reduced by 1, whereas that of a less deprived person (j) is increased by 1, so that the total scores in the two profiles are the same. However, the variance of the new profile ($\bar{y}$) is less than that of the original one ($\bar{x}$). Note that the rank preserving transformation in (10) does not alter the relative positions of the affected individuals and it reduces the deprivation score of the worse off person (i). This is the reason why we call it an *FCC*.

Marshall and Olkin (1979) defined a special kind of linear transformation, called a *T*-transformation, of a vector that leaves all but two components of the vector unchanged, and replaces these two components by averages. An *FCC* is a *T*-transformation, which is used extensively for studying dominance conditions, since:

(11)
$$\begin{aligned} \bar{y}_i &= \lambda \bar{x}_i + (1-\lambda)\bar{x}_j \\ \bar{y}_j &= (1-\lambda)\bar{x}_i + \lambda \bar{x}_j \\ \bar{y}_k &= \bar{x}_k \quad \text{for all } k \neq i, j, \end{aligned}$$

where $\lambda = \dfrac{(\bar{x}_i - \bar{x}_j - 1)}{(\bar{x}_i - \bar{x}_j)}$.

The following theorem gives an interesting consequence of the relation $\succeq_{SE}$ for additive exclusion measures that satisfy anonymity and have nondecreasing marginals.

Theorem 3: Let $x, y \in D^n$, where $\Sigma_{i=1}^n x_i = \Sigma_{i=1}^n y_i$. Then $x \succeq_{SE} y$ implies that $\Sigma_{i=1}^n h(x_i) \geq \Sigma_{i=1}^n h(y_i)$ for all individual exclusion measures h: $\mathbf{N}_0 \to \mathbf{R}$ whose marginals are nondecreasing.

Proof: Muirhead (1903) showed that given $x, y \in D^n$ along with $\Sigma_{i=1}^n x_i = \Sigma_{i=1}^n y_i$, if $x \succeq_{SE} y$ holds, then $\bar{y}$ can be derived form $\bar{x}$ by successive applications of a finite number of *FCC*s. Assume, without loss of generality, that only one *FCC* affecting individuals i and j, where $\bar{x}_i > \bar{x}_j$, takes us from $\bar{x}$ to $\bar{y}$.

Given $\bar{x}_i > \bar{x}_j$, let $\theta = \bar{x}_i - \bar{x}_j - 1$. Note that $\theta \in \mathbf{N}_0$. Since the marginal of the individual exclusion function h is nondecreasing, we have:

$$h(\bar{x}_j+1)-h(\bar{x}_j)\le h(\bar{x}_j+\theta+1)-h(\bar{x}_j+\theta), \tag{12}$$

which we can rewrite as:

$$h(\bar{x}_j+1)-h(\bar{x}_j)\le h(\bar{x}_i)-h(\bar{x}_i-1). \tag{13}$$

Inequality (13) on rearrangement gives:

$$h(\bar{x}_j+1)+h(\bar{x}_i-1)\le h(\bar{x}_i)+h(\bar{x}_j). \tag{14}$$

Substituting the values of $\bar{x}_j + 1$ and $\bar{x}_i - 1$ in (14), we get:

$$h(\bar{y}_j)+h(\bar{y}_i)\le h(\bar{x}_i)+h(\bar{x}_j). \tag{15}$$

Inequality (15) along with the information that $\bar{y}_k = \bar{x}_k$ for all $k \neq i, j$ gives us:

$$\sum_{l=1}^{n} h(\bar{y}_l)\le \sum_{l=1}^{n} h(\bar{x}_l). \tag{16}$$

Since the social exclusion measure $\Sigma h(.)$ satisfies anonymity, we can rewrite (16) as:

$$\sum_{l=1}^{n} h(y_l)\le \sum_{l=1}^{n} h(x_l), \tag{17}$$

which is the desired result.

Theorem 3 is very valuable. It shows how an *FCC* becomes helpful in ranking two exclusion profiles. It also provides a justification for using *NMS* as a postulate for a social exclusion measure.

In an *FCC* the deprivation scores of the two affected persons change in opposite directions. But often unidirectional changes in the scores of the two or more persons may take place. The following result, whose proof can be found in Fulkerson and Ryser (1962), states that under certain conditions the relation $x \succeq_{SE} y$, where the total scores in x and y are the same, is preserved.

Theorem 4: Let $x, y \in D^n$, *where* $\Sigma_{i=1}^{n} x_i = \Sigma_{i=1}^{n} y_i$, *be arbitrary. Then* $x \succeq_{SE} y$ *implies that* $(\bar{x} - e_j) \succeq_{SE} (\bar{y} - e_i)$, *where* $i \le j$ *and* e_k *is the n–coordinated vector with* 1 *in the* k^{th} *position and zeros elsewhere.*

The intuitive appeal of Theorem 4 is quite clear. Given that x dominates y if we reduce the degree of exclusion of one person in $\bar{x}$ and one person in $\bar{y}$, where the latter is relatively worse off than the former, the exclusion dominance remains preserved.

The following result, whose formal proof can be found in Fulkerson and Ryser (1962), is a generalization of Theorem 4.

Theorem 5: Let $x, y \in D^n$, where $\Sigma_{i=1}^n x_i = \Sigma_{i=1}^n y_i$, be arbitrary. Let u be obtained from $\bar{x}$ by reducing deprivation scores of persons in positions $i_1, i_2, \ldots, i_k$ by 1. Similarly, suppose v is obtained from $\bar{y}$ by reducing deprivation scores of persons in positions $j_1, j_2, \ldots, j_k$. If $i_1 \leq j_1, i_2 \leq j_2, \ldots, i_k \leq j_k$ and $x \succeq_{SE} y$, then $u \succeq_{SE} v$.

5. An Empirical Illustration

The purpose of this section is to illustrate the social exclusion measures proposed in this paper, namely: E_δ in (6), $\bar{E}$, the symmetric mean exclusion of order ν, and $\hat{E}$, the Gini exclusion measure using the European Community Household Panel (ECHP) data.[9] Note that the 14 non-monetary indicators defined below are based on subjective evaluations. Therefore, any definitional change or a change in the composition of a group will affect the analysis. Since this section can be regarded as an example of application of our indices, we take for granted the variables that Eurostat (2000) deemed appropriate to measure social exclusion. Since $\bar{E}$ and $\hat{E}$ are calculated to illustrate non-subgroup decomposability, we calculate them using population size dependent weights for different functionings. We base our analysis on the first six waves of ECHP, which cover the period from 1994 to 1999. The surveys are conducted nationally. The ECHP is an ambitious effort at collecting information on the living standards of the households of the EU member-states using common definitions, information collection methods and editing procedures. It contains detailed information on incomes, socio-economic characteristics, housing amenities, consumer durables, social relations, employment conditions, health status, subjective evaluation of well-being, etc. Of the 15 EU member-states, we could not consider Austria, Finland, Luxembourg, and Sweden since the data for these countries were not available for all the waves. For similar reasons we had to exclude Germany and the U.K. In particular, the ECHP surveys of these countries were substituted by national surveys, SOEP and BHPS respectively, that did not collect information on all the variables considered in our application.

Information was collected at the individual or the household level depending on the variable, but the unit of our analysis is the individual. The calculation uses required sample weights. In ECHP a person's quality of life has been measured along the following domains: financial difficulties, basic needs and consumption, housing conditions, durables, health, social contacts and participation, and life satisfaction. The 14 non-monetary indicators[10] suggested by Eurostat (2000) as best candidates to meet the following requirements are included in the analysis: (1) reflecting a negative aspect of a life pattern common to a majority of the population in the EU; (2) allowing international and intertemporal comparisons; and (3) expressing a link with income poverty. These are the following:

[9]Since our illustration involves cross-population comparisons, we drop the superscript n from E_δ^n, $\bar{E}^n$, and $\hat{E}^n$.

[10]In fact, the non-monetary indicators recommended in Eurostat (2000) are 15. We decided to drop the one belonging to the health domain, namely the proportion of people that were severely hampered in their daily activity by long-lasting health problems, since there was a considerable discontinuity between the ECHP waves for this indicator.

- Financial difficulties: 1. Persons living in households that have great difficulties in making ends meet. 2. Persons living in households that are in arrears with (re)payment of housing and/or utility bills.
- Basic necessities: 3. Persons living in households which cannot afford meat, fish or chicken every second day. 4. Persons living in households which cannot afford to buy new clothes. 5. Persons living in households which cannot afford a week's holiday away from home.
- Housing conditions: 6. Persons living in the accommodation without a bath or shower. 7. Persons living in dwellings with damp walls, floors, foundations, etc. 8. Persons living in households which have a shortage of space.
- Durables: 9. Persons not having access to a car due to lack of financial resources in the household. 10. Persons not having access to a telephone due to lack of financial resources in the household. 11. Persons not having access to a color TV due to lack of financial resources in the household.
- Health: 12. Persons (over 16) reporting bad or very bad health.
- Social contact: 13. Persons (over 16) who meet their friends or relatives less often than once a month (or never).
- Dissatisfaction: 14. Persons (over 16) being dissatisfied with their work or main activity.

While it is true that with a high income a person may be able to increase his purchasing power in several dimensions of well-being, low income should not be mixed up with falling short of minimum standards unambiguously in all dimensions. For instance, there is a debate about the importance of low income as a determinant of undernutrition (Lipton and Ravallion, 1995). In their illustration of the generalized human poverty index, Chakravarty and Majumder (2005) used the deprivations in three basic dimensions of life considered by UNDP (namely, failures in longevity, knowledge and decent living standard) and the anthropometric indicators, for example, "children with low birth weight," "undernourished people" and "children with low height for age." These dimensions of human life may not be mutually exclusive. Therefore, they carried a principal component analysis and the leading eigen value (which explains 69 percent of the total variance) puts weights ranging between 0.56 and 0.93 to the variables, thus justifying inclusion of all the variables in measuring the underlying latent construction of poverty. This parallels UNDP's arguments for including "adult literacy rate," "per capita real GDP" and "life expectancy at birth" in the construction of the human development index. That is why in this paper we include both financial difficulties and failures in other dimensions.

We first calculate E_δ for $\delta = 0$, 1, and 2 separately for two sets of indicators V_1 and V_2, where V_1 includes the indicators in the domains of financial difficulties, basic necessities, housing conditions, and durables, and V_2 includes the remaining indicators. The reason for separate calculations is that for indicators covered under V_1 we have household level information, whereas for the indicators in V_2 the available information is at the individual level, with the additional constraint that the minimum age of the reportee is 16. We prefer to keep the analysis separate and not to restrict the sample to V_2 since we do not want to exclude children from our data, who are considered in V_1 but not in V_2.

We call a person socially excluded with respect to a variable in a given domain if he has been deprived of the variable for at least four years out of the six years that we observe. In addition, exclusion for a functioning occurs if the person concerned is deprived for the last three years. Thus, our calculation of the individual exclusion score explicitly takes into account the dynamic or longitudinal aspect of social exclusion. A person's exclusion in a given domain has been obtained by adding up his exclusions over the concerned variables, that is, here the deprivation score is calculated under the assumption that $w_j = 1$ for all j.

Since in this calculation x_i is independent of the population size, *SUD* holds. Calculation of non-additive measures $\bar{E}$ and $\hat{E}$ involving x_i's which are dependent on the population size is presented later in the section. As an example of the construction of the individual exclusion scores, let's consider the variables in V_2: we assign value 0 to the individuals who had access to all the functionings in the relevant time period, 1 to those who had failure only in one dimension over the period, for instance, to those who met their friends or relatives less often than once a month (or never) or to those who were never satisfied with their main work or activity, and so on.

Numerical estimates of social exclusion for the EU member states are reported in Table 1. The upper part of the table presents the estimates for V_1 while its lower part gives the analogous values for V_2. The first column of the table gives the names of the countries for whom required information were available. In column 2 we report the population shares of different countries in the total of EU sample population considered for our analysis. In columns 3–5 we present, for each country, the values of E_δ for $\delta = 0$, 1 and 2 respectively.[11] The country-wise social exclusion levels are then weighted by the corresponding population shares to determine the contributions of different countries to total exclusion, which are given as percentages of total exclusion in columns 6–8. From a policy perspective, complete elimination of exclusion within a country would lower aggregate exclusion precisely by the percentage by which it contributes to total exclusion.

Several interesting features emerge from Table 1. We note that the values of measures as well as percentage contributions are sensitive to the values of δ. We first analyze the upper part of the table. Portugal turns out to be the country with the highest level of social exclusion, followed by Greece. But there is no unanimous agreement about the country with minimum exclusion. The Netherlands is the country with minimum H, whereas E_1 and E_2 regard Denmark as the country with the lowest level of social exclusion.

The maximum percentage contribution to total exclusion comes from Italy due to high exclusion scores and high population share, whereas Denmark is the least contributing country. Ireland and Belgium occupy respectively the second and third position in terms of low percentage contributions. The sixth column of this part of the table shows that Portugal, Italy, Spain, and Greece, the Southern European countries, report 68.13 percent of social exclusion as judged by the headcount index. Their contribution to overall exclusion rises to 71.69 percent (75.60 percent) if one uses A (E_2). The higher contributions of these four countries

[11]Recall that for $\delta = 0$ and 1, E_δ becomes respectively the head-count ratio, H, and the average deprivation score of the society, A.

TABLE 1

SOCIAL EXCLUSION IN EU MEMBER STATES (1993–98)

		Values of E_δ			Percentage Contributions Based on:		
	Population Shares	E_0 (head-count ratio, H)	E_1 (average deprivation score, A)	E_2	E_0 (head-count ratio, H)	E_1 (average deprivation score, A)	E_2
V_1							
Belgium	4.86	0.224	0.375	0.985	2.76	2.45	2.27
Denmark	2.54	0.195	0.273	0.495	1.26	0.93	0.60
Greece	4.98	0.592	1.605	6.235	7.47	10.75	14.71
Spain	18.91	0.510	0.897	2.202	24.45	22.83	19.74
France	27.35	0.332	0.549	1.317	23.00	20.21	17.08
Ireland	1.72	0.359	0.749	2.336	1.56	1.73	1.90
Italy	27.46	0.397	0.668	1.627	27.65	24.69	21.18
Netherlands	7.35	0.177	0.301	0.734	3.29	2.98	2.56
Portugal	4.82	0.700	2.067	8.731	8.56	13.42	19.97
Total	100	0.394	0.743	2.110	100	100	100
V_2							
Belgium	4.82	0.061	0.063	0.068	2.90	2.67	2.31
Denmark	2.56	0.034	0.035	0.037	0.85	0.78	0.67
Greece	5.09	0.074	0.078	0.087	3.72	3.48	3.10
Spain	18.70	0.087	0.088	0.091	15.95	14.38	11.90
France	26.84	0.082	0.092	0.115	21.55	21.58	21.51
Ireland	1.55	0.022	0.023	0.025	0.34	0.31	0.27
Italy	28.25	0.146	0.172	0.23	40.52	42.33	45.51
Netherlands	7.42	0.032	0.034	0.039	2.31	2.21	2.04
Portugal	4.76	0.254	0.295	0.381	11.86	12.25	12.69
Total	100	0.102	0.115	0.143	100	100	100

Notes:

V_1 considers jointly the variables included in the domains of financial difficulties, basic necessities, housing conditions, durables.

V_2 considers jointly the variables included in the domains of health, social contact and dissatisfaction.

Estimates derived using distributions of persons, with the additional constraint of age being at least 16 for V_2.

is partly due to their almost average or more than average social exclusions. Spain and France come next after Italy in the ranking by percentage contributions. A comparison between Italy and Ireland is worth noting here. Although the latter has a better position than the former with respect to H and A, for the other measure it becomes worse off. The reason is that the variance of the deprivation scores is much higher in Ireland than in Italy. By percentage contributions, Ireland shows a much better picture than Italy. This is because the country has a very low population share among the member states.

In V_2 as well, Portugal is the member state with the highest level of social exclusion and Italy by percentage contribution. Ireland performs the best by showing the lowest values with respect to both the factors. Belgium, Denmark and the Netherlands also show low values for both factors. But Denmark has a better position than the other two countries by percentage contributions, and Denmark and the Netherlands perform better than Belgium by the other factor. France and Spain do not have unambiguous ranking between themselves with respect to index values, but by percentage contributions France is regarded as worse than Spain. These two countries perform worse than Greece by both the factors. Portugal, Italy, Spain and France jointly contribute more than 87 percent to total exclusion by any measure. Finally, except for Portugal, the ranking of countries by any measure in V_2 is different from that in V_1.

From a policy point of view, the breakdown of the variables into two subgroups enables us to identify the countries separately in each subset that are most susceptible to exclusion.

In Table 2 we carry out a similar analysis for Italy. The country has been divided into 11 geographic areas.[12] In V_1, the South is the area with the highest level of social exclusion by E_1 and E_2, while Sardegna occupies this position for H. Similarly, there is no unanimous agreement about the area with the lowest level of social exclusion. It is worth noting that South is only a part of the south of the country. If we add to South the remaining southern area, namely Campania, we can conclude that the southern areas contribute between 33 and 46 percent to total exclusion observed in Italy, depending on the measure. We note the difference with the northern regions, namely North–West, North–East, Lombardia and Emilia–Romagna, whose total percentage contribution ranges between 14 and 25 percent. The other two areas with high levels of exclusion are the two islands, Sicilia and Sardegna, while only the former presents high percentage contributions. In the same way in V_2, South is the geographic area with the highest level of social exclusion and unanimous agreement about the area with minimum exclusion is not reached. However, the northern areas occupy low exclusion positions without showing unambiguous ranking among themselves. More generally, ranking of areas by any measure is different in V_1 to that in V_2.

The high contributing areas require attention from a policy perspective for reduction of their contributions so that a higher living standard can be achieved.

In Table 3 we present results of deprivation scores using population size dependent weights. The measures that we apply are $\bar{E}$, the symmetric mean

[12]The information on the geographic areas of the Italian households are available in ECHP at the Nuts 1 level.

TABLE 2

SOCIAL EXCLUSION IN ITALY BY GEOGRAPHIC AREAS (1993–98)

		Values of E_δ			Percentage Contributions Based on:		
	Population Shares	E_0 (head-count Ratio, H)	E_1 (average deprivation score, A)	E_2	E_0 (head-count ratio, H)	E_1 (average deprivation score, A)	E_2
V_1							
North West	10.65	0.210	0.318	0.626	5.64	5.07	4.10
Lombardia	15.39	0.236	0.317	0.551	9.13	7.31	5.21
North East	11.33	0.227	0.313	0.543	6.48	5.30	3.78
Emilia-Romagna	7.06	0.223	0.249	0.317	3.96	2.63	1.38
Centre	10.39	0.402	0.591	1.192	10.51	9.19	7.61
Lazio	9.02	0.390	0.620	1.409	8.85	8.37	7.80
Abruzzo-Molise	2.84	0.434	0.580	0.961	3.10	2.47	1.68
Campania	9.88	0.541	0.947	2.347	13.45	14.00	14.24
South	11.84	0.666	1.460	4.454	19.86	25.86	32.40
Sicilia	8.68	0.644	1.108	3.054	14.08	14.40	16.30
Sardegna	2.92	0.670	1.239	3.071	4.93	5.42	5.51
Total	100	0.397	0.668	1.627	100	100	100
V_2							
North West	11.06	0.105	0.125	0.178	7.96	8.06	8.56
Lombardia	15.75	0.090	0.107	0.150	9.65	9.85	10.24
North East	11.38	0.098	0.112	0.143	7.65	7.44	7.07
Emilia-Romagna	7.29	0.122	0.130	0.146	6.05	5.50	4.63
Centre	10.46	0.164	0.188	0.241	11.70	11.42	10.95
Lazio	8.76	0.139	0.166	0.227	8.31	8.48	8.63
Abruzzo-Molise	2.79	0.140	0.167	0.220	2.68	2.71	2.67
Campania	9.31	0.212	0.248	0.329	13.49	13.44	13.28
South	11.60	0.242	0.293	0.407	19.22	19.78	20.50
Sicilia	8.56	0.155	0.187	0.263	9.07	9.32	9.78
Sardegna	3.03	0.204	0.227	0.280	4.22	4.00	3.68
Total	100	0.146	0.172	0.230	100	100	100

Notes:

V_1 considers jointly the variables included in the domains of financial difficulties, basic necessities, housing conditions, durables.

V_2 considers jointly the variables included in the domains of health, social contact and dissatisfaction.

Estimates derived using distributions of persons, with the additional constraint of age being at least 16 for V_2.

TABLE 3

SOCIAL EXCLUSION IN EU MEMBER STATES (1993–98)

	Values of E_δ			
	$\bar{E}$ (v = 0.5)	$\bar{E}$ (v = 1)	$\bar{E}$ (v = 2)	$\hat{E}$
V_1				
Belgium	14.072	15.598	19.826	34.283
Denmark	12.460	13.192	15.155	19.056
Greece	17.486	19.829	24.607	36.615
Spain	12.100	13.519	16.936	27.222
France	13.485	14.780	18.199	25.966
Ireland	16.907	19.059	23.830	61.128
Italy	11.266	16.295	12.677	35.034
Netherlands	15.729	17.050	20.386	43.573
Portugal	17.084	19.834	25.327	74.395
V_2				
Belgium	10.293	10.361	10.568	13.333
Denmark	10.282	10.338	10.495	12.488
Greece	10.435	10.530	10.809	12.449
Spain	9.261	9.293	9.391	11.777
France	11.056	11.267	11.833	13.398
Ireland	10.295	10.353	10.516	16.781
Italy	10.471	10.758	11.526	18.312
Netherlands	10.623	10.756	11.136	14.098
Portugal	9.734	10.015	10.724	28.515

Notes:

V_1 considers jointly the variables included in the domains of financial difficulties, basic necessities, housing conditions, durables.

V_2 considers jointly the variables included in the domains of health, social contact and dissatisfaction.

Estimates derived using distributions of persons, with the additional constraint of age being at least 16 for V_2.

exclusion of order v, and $\hat{E}$, the Gini exclusion measure. Here we take into account the local dimension of the concept, i.e. people compare themselves with their reference society, and following Runciman (1966), we define the degree of deprivation inherent in not having access to an item as an increasing function of the proportion of persons in the society who have access to the item. Hence the weight attached to attribute j, w_j, reflects the percentage of the population in the country of residence of the individual that is not deprived from that specific attribute. More precisely, we assume that, if the percentage of the population not deprived of functioning j lies in the interval $(10\,(i-1), 10i]$, where $i = 1, 2, \ldots, 10$, then $w_j = i$. If nobody is excluded from j, then the definition of the characteristic function ensures that deprivation with respect to j is zero.

The upper part of the table presents the estimates for V_1 while its lower part gives the analogous values for V_2. In columns 2–4 we present, for each country, the values of $\bar{E}$, for $v = 0.5$, 1 and 2 respectively. The parameter v is the sensitivity parameter; the more positive it is, the more sensitive the index will be to the capability failures of the more deprived. In column 5 the values of the Gini exclusion measure, $\hat{E}$, are reported.

The results are strikingly different from the analysis of Table 1 in the case of both V_1 and V_2. The reason behind this is that in the case of Table 1 for all countries we use constant weights in order to calculate deprivation scores of a

person, whatever the proportions of population that are better off than him in the relevant dimensions. In contrast, Table 3 is based on Runciman-type weights for deprivation scores that explicitly take into account the population size of a country, that is, the weights are country-wise population size-specific. South European countries are split into two groups located at the opposite side of the ranking with respect to $\bar{E}$, due to the weighting scheme reflecting on an average higher percentage of the population deprived in Portugal and Greece than in Spain and Italy. On the one hand, Portugal and Greece are still the most deprived countries, while Spain and Italy now with Denmark are the countries where social exclusion is lowest. The latter is also the country with minimum exclusion according to the Gini measure. When we consider relatively high exclusion values (more than 35), starting with Italy the ranking of countries from low to high exclusion by the Gini measure is Italy, Greece, the Netherlands, Ireland and Portugal. Another notable difference with the previous unweighted case is that of the Netherlands. It is now a member state with a relatively high level of social exclusion according to all the measures. In the Netherlands the percentage of the population deprived in all dimensions is low, reflecting high weights assigned to those who are deprived; in addition there is more cumulation of disadvantage since the excluded individuals are more likely to be so in more dimensions at the same time.

For V_2, the domains of health, social contact and dissatisfaction, the values of $\bar{E}$ are quite similar among all the countries, while we observe more variance for $\hat{E}$. The lowest excluded country by $\bar{E}$ is always Spain, followed by Portugal when $\nu = 0.5$ and 1, and Denmark when $\nu = 2$. On the contrary, Portugal is the most excluded country when disadvantage is evaluated with the Gini measure, while France is the country with the highest level of exclusion by $\bar{E}$.

6. Conclusions

Social exclusion refers to inability of a person to participate in basic day-today economic and social activities of life.

In this paper we have developed an axiomatic approach to the measurement of social exclusion and characterized the class of subgroup decomposable measures of exclusion. We have also proposed nondecomposable measures that could be applied to take into account the local dimension of the concept. A dominance criterion for ranking two societies by symmetric additive exclusion measures under constancy of population size and total deprivation score was suggested. An application of the decomposable and nondecomposable measures considered in the paper has been made using European Union data.

Several extensions of our analysis are possible. First, a characterization of some class of measures, for example of E_δ, will be quite interesting. Second, extension of our dominance criterion to the cases of nonadditive measures, variable total and variable population size, and a rigorous discussion on the converse of Theorems 3 and 4 will be worthwhile. Finally, we have considered only decomposability according to population subgroups. We can as well consider decomposition of population exclusion by attributes and study the impact of each of them on the aggregate exclusion. This will enable us to identify the attributes that are more/less susceptible to social exclusion.

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