INDIAN STATISTICAL INSTITUTE

ENDTERM EXAMINATION
M.TECH(CS/CrS) & M. MATH II YEAR

AUTOMATA THEORY, LANGUAGES AND COMPUTATION

Date: 24.11.2025 Maximum marks: 100 Duration: 3.0 hours.

The paper contains 110 marks. Answer as much as you can, the maximum you can score is 100.

1. Consider the following class of languages numbered **Class 1** to **Class 7**.

   Class 1: Empty

   Class 2: Finite

   Class 3: Regular

   Class 4: Context Free

   Class 5: Recursive

   Class 6: Recursively Enumerable

   Class 7: All Languages

   For the listed languages below, what is the lowest numbered class to which the language surely belongs. For example, the correct classification for a context free language which is not regular would be class **4** (though it also belongs to all the classes with numbers **4** and above. Similarly, suppose a language L is known to be recursively enumerable and it could possibly be recursive but the available information do not guarantee that it is recursive, then L should be classified into **Class 6** .
   You just need to write the class number against the following languages, no justification is required.

   (a) A language in **coNP**.

   (b) The complement of a nonrecursive language.

   (c) The complement of a context free language.

   (d) The intersection of a recursive language and a recursively enumerable language.

   (e) The intersection of a recursive language and a language which is not recursively enumerable.

   $[5 \times 2 = 10]$

2. State if the following statements are TRUE or FALSE. Give a short argument in favor of your answer.

(a) Every subset of a regular language is regular.

(b) The class of non-regular languages is closed under complementation.

(c) Let $\equiv$ be any right congruence of finite index on Σ^*. Any equivalence class of $\equiv$ is a regular language.

(d) $L(M)$ is recursive if and only if the Turing machine M is total.

(e) It is known that there is no polynomial time algorithm to decide $\overline{\mathsf{SAT}}$.

(f) If B is recursively enumerable and $A \leq_p B$ then A is recursively enumerable.

(g) If A is recursive and $A \leq_p B$ then B must be recursive.

(h) If A is **NP**-complete and $A \leq_p B$ then B must be **NP**-complete.

(i) Suppose $L_1 \in \mathbf{P}$ and L_2 is **NP**-complete, then it is known that there cannot be a polynomial time reduction from L_2 to L_1.

(j) If $A \leq_p B$, then it is possible that A is in **P** and **B** is not in **NP**.

[$10 \times 2 = 20$]

3. (a) You are given two DFAs A and B. Describe the construction of a DFA M, such that

$$L(M) = (L(A) - L(B)) \cup (L(B) - L(A))$$

(b) Prove that given a DFA M which accepts $L \subseteq \Sigma^*$ there exists a Myhill Nerode relation $\equiv_M$ on Σ^* for L.

[10+10 = 20]

4. (a) Show that the language $L_1 = \{(0^k 1^k)^k : k \geq 1\}$ is not a CFL.

(b) Is the language $L_2 = \{0^m 1^n : m \neq n\}$ a CFL? If L_2 is a CFL, give a CFG for it.

[10+10 = 20]

5. (a) Consider the following languages A and B:

$$\begin{aligned} A &= \{(M, w) : w \in L(M)\} \\ B &= \{M : L(M) \text{ is finite}\} \end{aligned}$$

Prove that $A \leq_m B$.

(b) Let $L = \{(M_1, M_2) : L(M_1) = L(M_2)\}$, i.e., L contains encodings of two Turing Machines M_1, M_2, such that the language accepted by M_1 and M_2 are the same. Prove that L is not recursive by using a reduction from any known non-recursive language.

[10+10 = 20]

6. (a) Let P be a property of recursively enumerable sets. When is P said to be monotone?

(b) Let P be a non monotone property of recursively enumerable sets and let $T_P = \{M : P(L(M)) = T\}$. Prove that T_P is not recursively enumerable. You can assume $\overline{HP}$ to be not recursively enumerable.

[5+15 = 20]