Indian Statistical Institute
Second Semester Examinations: 2024-25

Course Name: M. Math, 2^{nd} year
Subject Name : Topology III
Maximum Marks: 50, Duration: Three hours
Date: 19.11.2025, 2:30 PM – 5:30 PM

- Answer as many questions as you can.
- Maximum marks is 50.
- You may use any results proved in class. Any other results (including those in homework problem sets) require proof.

1. Prove that every complex line bundle over S^2 is isomorphic to $\gamma^{\otimes k}$ or $(\gamma^*)^{\otimes k}$ for some $k \geq 0$. (Here, γ denotes the canonical line bundle over $\mathbb{C}P^1 \cong S^2$.) 8

2. Prove that $\Omega\mathbb{H}P^n \simeq S^3 \times \Omega S^{4n+3}$. 8

3. Prove that $(S^1 \times \mathbb{C}P^\infty)/(S^1 \times *)$ is not weakly equivalent to $S^3 \times \mathbb{C}P^\infty$. 10

4. a) Compute the homology groups of $Gr_2(\mathbb{C}^4)$, the (complex) Grassmannian of 2-planes in $\mathbb{C}^4$. 5
   b) Show that the map $Gr_2(\mathbb{C}^4) \to Gr_2(\mathbb{C}^\infty) \simeq BU(2)$ is a 5-equivalence. 5

5. Prove that as cohomology operations, $Sq^4 \neq Sq^3 \circ Sq^1$. 8

6. Prove that $U(3)$ is not weakly equivalent to $S^1 \times S^3 \times S^5$ as topological spaces. 10